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Asymmetries in polarized hadron production in e^+e^- annihilation up to order $1/Q$

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Abstract

We present the results of the tree-level calculation of inclusive two-hadron production in electron–positron annihilation via one photon up to subleading order in $1/Q$. We consider the situation where the two hadrons belong to different, back-to-back jets. We include polarization of the produced hadrons and discuss azimuthal dependences of asymmetries. New asymmetries are found, in particular there is a *leading* $\cos(2\phi)$ asymmetry, which is even present when hadron polarization is absent, since it arises solely due to the intrinsic transverse momenta of the quarks.
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1. Introduction

Three basic hard scattering processes in which the structure of hadrons is studied with electroweak probes are (semi-)inclusive lepton–hadron scattering, the Drell–Yan process and inclusive hadron production in e^+e^- annihilation. In this paper we will focus on the latter, where we restrict ourselves to photon exchange. The (timelike) photon momentum q sets the scale Q , where $Q^2 \equiv q^2$, which is much larger than the characteristic hadronic scales.

The inclusive lepton–hadron scattering, generally known as deep inelastic scattering (DIS), is the most studied process from the theoretical as well as from the experimental side. From the theoretical point of view DIS can be described by using the operator product expansion (OPE), within the context of quantum chromodynamics (QCD).

This allows us to relate moments of structure functions to (Fourier transforms of) hadronic matrix elements of local operators. In the (QCD improved) parton model, i.e. to leading order in $1/Q$, the structure functions can be expressed as sums of so-called distribution functions (DFs). The OPE thus gives information on *moments* of the DFs. However, one can also describe a scattering process directly in terms of the DFs themselves, which are (Fourier transforms of) hadronic matrix elements of non-local operators [1,2]. For treating the non-leading orders in $1/Q$, Ellis et al. (EFP) [3] have developed a formalism for unpolarized DIS, using non-local operators, following ideas by Politzer [4]. The extension to polarized DIS was done by Efremov and Teryaev [5]. At tree level, i.e. order $(\alpha_s)^0$, EFP have shown the equivalence of their formalism to the OPE approach for $(1/Q)^n$ power corrections.

The non-local operators consist of quark and gluon fields. The quark (gluon) DFs are functions of the light-cone momentum fraction $x = p^+/P^+$ of a quark (gluon) with momentum p in a hadron with momentum P . For multi-parton DFs, which show up beyond leading order, the functions depend on momentum fractions of several partons [4,6]. Often the number of partons can be reduced with help of the QCD equations of motion.

The EFP approach was extended to other processes in analogy to the DIS description, in particular, to those ones where the OPE is not applicable (for an alternative approach based on a non-local operator expansion see Ref. [7]). The Drell–Yan process has been studied to leading order by Ralston and Soper [8] and partially to subleading order in Refs. [9–11]. Here the cross section is a sum of products of two DFs. Recently, the complete tree-level result up to order $1/Q$ for semi-inclusive polarized lepton–hadron scattering was published [12]. In this case one needs to include so-called fragmentation functions (FFs) [13,2] and the cross section is a sum of products of a DF and a FF.

The present paper focuses on the third process of interest: inclusive two-hadron production in e^+e^- annihilation up to order $1/Q$, where the two hadrons belong to different, back-to-back jets. The cross section involves products of FFs, the number of which is larger than the number of DFs, due to the appearance of so-called time-reversal-odd structures, which arise since time reversal does not give constraints in this case. Interesting features like the Collins effect [14] show up, as do other new asymmetries.

The DFs and FFs are essentially non-perturbative objects and must be determined by experiment or calculated, for instance with the help of models. The idea is that these functions are universal and once they are measured in one process they can lead to predictions in others. The DFs and FFs parametrize the structure of a hadron, e.g. the spin structure, and the leading order DFs and FFs have simple probabilistic interpretations. With the help of symmetry properties one can first of all determine the set of DFs and FFs in which a process can be expressed. By giving cross sections and asymmetries in terms of these functions one can deduce how to measure and separate them.

From the experimental side most well known are the leading order unpolarized and polarized DFs called f_1 and g_1 , respectively [15] and the FF D_1 [16]. For subleading order the DF g_2 has been measured [17], but still with rather large errors. The experimental knowledge on FFs is much smaller than that on DFs. We obtain the most general expression for the cross section in terms of as yet unknown FFs, in order to find out

which other functions might be experimentally accessible in the near future and where to look for them. Some of the unknown functions have been modeled [18], the results of which can be used to estimate the magnitudes of asymmetries from our expressions.

Some asymmetries, like the Collins effect, are leading effects, not suppressed by powers of $1/Q$, but one needs to include intrinsic transverse momentum in the DFs and FFs [1,8,2]. Intrinsic transverse momentum plays a crucial role in two-hadron processes involving two soft, non-perturbative, parts, since the transverse momenta are linked by momentum conservation. Those DFs or FFs which, as functions of transverse momentum, would not contribute in one-hadron processes, will show up in these two-hadron processes. The idea that intrinsic transverse momentum always gives rise to suppression is incorrect, although it is the case in one-hadron processes. In the pioneering work [19–21] on azimuthal dependences due to intrinsic transverse momentum, all effects are found to be suppressed by at least $1/Q$.

On the formal theoretical side there remains the open issue of how to proof factorization for the process under consideration. For the case of back-to-back jets there exists a proof, but no higher twist effects are included [22]. We do not expect polarization to be a problem for factorization (see Ref. [23]). For the case of the Drell–Yan process, which is very similar to the process under consideration, arguments have been given why factorization holds for the first non-leading power corrections [24,25], which is consistent with its known failure at order $1/Q^4$ [26–28]. We expect factorization to hold also for the case at hand.

We will not concern ourselves with these problems here, even though factorization is needed to ensure universality of the FFs, and restrict the discussion to tree level. It represents the extension of the naive parton model to subleading order and shows the dominant structures to be expected in the cross section, although QCD corrections, such as Sudakov effects [29], may affect the magnitude of the asymmetries. At tree level the only QCD input at order $1/Q$ (apart from the Feynman rules) is the use of the equations of motion which ensures the electromagnetic gauge invariance.

The outline of this paper is as follows. In Section 2 we present the formalism of the e^+e^- annihilation process, with emphasis on the kinematics. Section 3 contains the analysis of the soft parts of the process, in particular the fragmentation functions are studied. This is followed by the details of the complete tree-level calculation of the hadron tensor up to subleading order in Section 4, the result of which is given in Appendix B. In the three following sections we investigate special cases, which give more insight than the full result and are useful from a practical point of view. In Section 5 we discuss the result after integration over the transverse momentum of the photon. In Section 6 we study the differential cross section, i.e. not integrated over transverse photon momentum, but restricted to leading order and the case where only one hadron is polarized (the case of two polarized hadrons is given in Appendix C). In Section 7 this is compared with the integrated cross section weighted with factors of the transverse momentum of the photon. Finally, the results are summarized in Section 8.

2. Kinematics

We consider $e^- + e^+ \rightarrow$ hadrons, where the two leptons with momenta l and l' annihilate into a photon with momentum $q = l + l'$, which is timelike with $q^2 \equiv Q^2 \rightarrow \infty$. Denoting the momentum of outgoing hadrons by P_h ($h = 1, 2, \dots$) we use invariants $z_h = 2P_h \cdot q / Q^2$. The momenta can also be considered as jet momenta. We will consider the general case of polarized leptons with helicities $\pm\lambda_e$ and production of hadrons of which the spin states are characterized by a spin vector S_h ($h = 1, 2, \dots$), satisfying $S_h^2 = -1$ and $P_h \cdot S_h = 0$. In this way we can treat the case of unpolarized final states or final state hadrons with spin 0 and spin $\frac{1}{2}$. We will work in the limit where Q^2 and $P_h \cdot q$ are large, keeping the ratios z_h finite.

The square of the amplitude can be split into a purely leptonic and a purely hadronic part,

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}, \quad (1)$$

with the helicity-conserving lepton tensor (neglecting the lepton masses) given by

$$L_{\mu\nu}(l, l'; \lambda_e) = 2l_\mu l'_\nu + 2l_\nu l'_\mu - Q^2 g_{\mu\nu} + 2i\lambda_e \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma. \quad (2)$$

For the case of two observed hadrons in the final state, the product of hadronic current matrix elements is written as

$$H_{\mu\nu}(P_X; P_1 S_1; P_2 S_2) = \langle 0 | J_\mu(0) | P_X; P_1 S_1; P_2 S_2 \rangle \langle P_X; P_1 S_1; P_2 S_2 | J_\nu(0) | 0 \rangle, \quad (3)$$

where a summation over spins of the unobserved *out* state is understood. The cross section (including a factor $\frac{1}{2}$ from averaging over incoming polarizations) is: For *two-particle inclusive* $e^+ e^-$ annihilation

$$\frac{P_1^0 P_2^0 d\sigma^{(e^+ e^-)}}{d^3 P_1 d^3 P_2} = \frac{\alpha^2}{4Q^6} L_{\mu\nu} \mathcal{W}^{\mu\nu}, \quad (4)$$

with

$$\begin{aligned} \mathcal{W}_{\mu\nu}(q; P_1 S_1; P_2 S_2) &= \frac{1}{(2\pi)^4} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q - P_X - P_1 - P_2) \\ &\quad \times H_{\mu\nu}(P_X; P_1 S_1; P_2 S_2); \end{aligned} \quad (5)$$

For *one-particle inclusive* $e^+ e^-$ annihilation

$$P_h^0 \frac{d\sigma}{d^3 P_h} = \frac{\alpha^2}{2Q^6} L_{\mu\nu} W^{\mu\nu}, \quad (6)$$

with

$$\begin{aligned} W_{\mu\nu}(q; P_h S_h) &= \frac{1}{(2\pi)} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q - P_X - P_h) \langle 0 | J_\mu(0) | P_X; P_h S_h \rangle \\ &\quad \times \langle P_X; P_h S_h | J_\nu(0) | 0 \rangle, \end{aligned} \quad (7)$$

and for the *totally inclusive* annihilation cross section the well-known result

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi^2\alpha^2}{Q^6} L_{\mu\nu} R^{\mu\nu}, \quad (8)$$

with the tensor $R_{\mu\nu}$ given by

$$\begin{aligned} R_{\mu\nu}(q) &= \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q - P_X) \langle 0 | J_\mu(0) | P_X \rangle \langle P_X | J_\nu(0) | 0 \rangle \\ &= \int d^4 x e^{iq \cdot x} \langle 0 | [J_\mu(x), J_\nu(0)] | 0 \rangle. \end{aligned} \quad (9)$$

Recall that the totally inclusive cross section is directly related to the vacuum polarization. Also note that the totally inclusive process is short-distance dominated, whereas the one-particle inclusive case is light-cone dominated, but only in the former the OPE can be applied.

In order to expand the lepton and hadron tensors in terms of independent Lorentz structures, it is convenient to work with vectors orthogonal to q . A normalized timelike vector is defined by q and a normalized spacelike vector is defined by $\tilde{P}^\mu = P^\mu - (P \cdot q/q^2)q^\mu$ for one of the outgoing momenta, say P_2 ,

$$\hat{t}^\mu \equiv \frac{q^\mu}{Q}, \quad (10)$$

$$\hat{z}^\mu \equiv \frac{Q}{P_2 \cdot q} \tilde{P}_2^\mu = 2 \frac{P_2^\mu}{z_2 Q} - \frac{q^\mu}{Q}. \quad (11)$$

Note that we have neglected $1/Q^2$ corrections, as we will do throughout the paper. Such corrections arise among others from the hadron masses M_h , so-called target mass corrections or kinematic power corrections.

Vectors orthogonal to \hat{z} and \hat{t} are obtained with the help of the tensors

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} - \hat{t}^\mu \hat{t}^\nu + \hat{z}^\mu \hat{z}^\nu, \quad (12)$$

$$\epsilon_\perp^{\mu\nu} \equiv -\epsilon^{\mu\nu\rho\sigma} \hat{t}_\rho \hat{z}_\sigma = \frac{1}{(P_2 \cdot q)} \epsilon^{\mu\nu\rho\sigma} P_{2\rho} q_\sigma. \quad (13)$$

For instance, using the other hadronic momentum P_1 , one obtains $P_{1\perp}^\mu = g_\perp^{\mu\nu} P_{1\nu}$ (see Fig. 1). We define the normalized vector $\hat{h}^\mu = P_{1\perp}^\mu / |P_{1\perp}|$ and the second orthogonal direction is given by $\epsilon_\perp^{\mu\nu} \hat{h}_\nu$. We use boldface vectors to denote the two-dimensional Euclidean part of a four-vector, such that $P_{1\perp} \cdot P_{1\perp} = -P_{1\perp} \cdot P_{1\perp}$.

In the calculation of the hadron tensor it will be convenient to define lightlike directions using the hadronic (or jet) momenta. Consider two hadronic momenta P_1 and P_2 not belonging to one jet (i.e. their dot product $P_1 \cdot P_2$ is of order Q^2). The momenta can then be parametrized using dimensionless lightlike vectors n_+ and n_- satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$,

$$P_1^\mu \equiv \frac{\zeta_1 \tilde{Q}}{\sqrt{2}} n_-^\mu + \frac{M_1^2}{\zeta_1 \tilde{Q} \sqrt{2}} n_+^\mu, \quad (14)$$

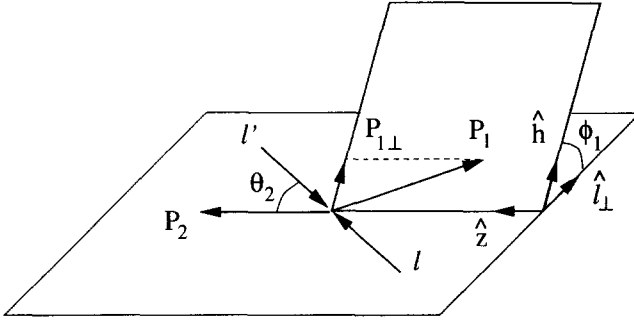


Fig. 1. Kinematics of the annihilation process in the lepton center-of-mass frame for a back-to-back jet situation. P_2 is the momentum of a jet or of a fast hadron in a jet, P_1 is the momentum of a hadron belonging to the other jet.

$$P_2^\mu \equiv \frac{M_2^2}{\zeta_2 \tilde{Q} \sqrt{2}} n_-^\mu + \frac{\zeta_2 \tilde{Q}}{\sqrt{2}} n_+^\mu, \quad (15)$$

$$q^\mu \equiv \frac{\tilde{Q}}{\sqrt{2}} n_-^\mu + \frac{\tilde{Q}}{\sqrt{2}} n_+^\mu + q_T^\mu, \quad (16)$$

where $\tilde{Q}^2 = Q^2 + Q_T^2$ with $q_T^2 \equiv -Q_T^2$. We will use the notation $p^- = p \cdot n_+$ and $p^+ = p \cdot n_-$ for a generic momentum p . For the case of two back-to-back jets $Q_T^2 \ll Q^2$ and up to Q_T^2/Q^2 , which we neglect, one has $\tilde{Q} = Q$, $\zeta_1 = z_1$ and $\zeta_2 = z_2$. If the momentum P_2 is used to define the vector \hat{z}^μ , then

$$P_{1\perp}^\mu = -z_1 q_T^\mu. \quad (17)$$

Vectors transverse to n_+ and n_- one obtains using the tensors

$$g_T^{\mu\nu} \equiv g^{\mu\nu} - n_+^{(\mu} n_-^{\nu)}, \quad (18)$$

$$\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma}, \quad (19)$$

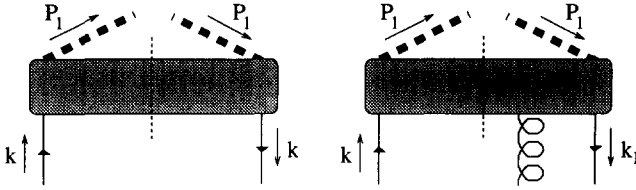
where the brackets around the indices indicate symmetrization. Note that these *transverse* tensors are not identical to the *perpendicular* ones defined above if the transverse momentum of the outgoing hadron does not vanish. The lightlike directions, however, can easily be expressed in \hat{t} , \hat{z} and a perpendicular vector,

$$n_+^\mu = \frac{1}{\sqrt{2}} [\hat{t}^\mu + \hat{z}^\mu], \quad (20)$$

$$n_-^\mu = \frac{1}{\sqrt{2}} \left[\hat{t}^\mu - \hat{z}^\mu - 2 \frac{q_T^\mu}{Q} \right] = \frac{1}{\sqrt{2}} \left[\hat{t}^\mu - \hat{z}^\mu - 2 \frac{Q_T}{Q} \hat{h}^\mu \right], \quad (21)$$

showing that the differences are of order $1/Q$. Especially for the treatment of azimuthal asymmetries, it is important to keep track of these differences.

In summary, we use two sets of basis vectors, the first set constructed from the photon momentum (q) and one of the hadron momenta (P_2), the second set from the two hadron momenta (P_1 and P_2). The respective frames where the momenta q and P_2 ,

Fig. 2. Correlation functions Δ and Δ_A .

or P_1 and P_2 , are collinear are the natural ones connected to these two sets. In the first P_1 has a perpendicular component $P_{1\perp}$, in the second q has a transverse component q_T . One can get from one frame to the other via a Lorentz transformation that leaves the minus components unchanged [30].

3. Correlation functions

In this section we discuss the relevant “soft” hadronic matrix elements that appear in the diagrammatic expansion of a hard scattering amplitude. Assuming the two hadrons to belong to two different jets we encounter two types of soft parts in the process under consideration: one describes the fragmentation of a quark into a hadron plus a remainder which is not detected and the other describes the similar fragmentation for an antiquark. Up to order $1/Q$ the quark fragmentation is described with the help of two types of correlation functions: the quark–quark correlation function $\Delta(P_1, S_1; k)$ [2] and the quark–gluon–quark correlation function $\Delta_A^\alpha(P_1, S_1; k, k_1)$ (Fig. 2),

$$\Delta_{ij}(P_1, S_1; k) = \sum_X \frac{1}{(2\pi)^4} \int d^4x e^{ik \cdot x} \langle 0 | \psi_i(x) | P_1, S_1; X \rangle \langle P_1, S_1; X | \bar{\psi}_j(0) | 0 \rangle, \quad (22)$$

$$\begin{aligned} \Delta_{Aij}^\alpha(P_1, S_1; k, k_1) = & \sum_X \frac{1}{(2\pi)^4} \int d^4x d^4y e^{ik \cdot y + ik_1 \cdot (x-y)} \\ & \times \langle 0 | \psi_i(x) g A_T^\alpha(y) | P_1, S_1; X \rangle \langle P_1, S_1; X | \bar{\psi}_j(0) | 0 \rangle, \end{aligned} \quad (23)$$

where k, k_1 are the quark momenta and an averaging over color indices is understood. If one chooses the gauge $A^- = 0$ only a transverse gluon is relevant. In fact, in a calculation up to subleading order, we only encounter the partly integrated correlation functions $\int dk^+ \Delta(P_1, S_1; k)$ and $\int dk^+ d^4k_1 \Delta_A^\alpha(P_1, S_1; k, k_1)$, which are functions of k^- and k_T only. Note that the definition of Δ_A^α includes one power of the strong coupling constant g .

The above matrix elements as functions of invariants are assumed to vanish sufficiently fast above a characteristic hadronic scale, which is much smaller than Q^2 . This means that in the above matrix elements $k^2, k \cdot P_1 \ll Q^2$. Hence, we make the following Sudakov decomposition for the quark momentum k :

$$k \equiv \frac{z_1 Q}{z\sqrt{2}} n_- + \frac{z(k^2 + k_T^2)}{z_1 Q\sqrt{2}} n_+ + k_T \approx \frac{1}{z} P_1 + k_T. \quad (24)$$

Similarly, we decompose the spin vector S_1 ,

$$S_1 \equiv \frac{\lambda_1 z_1 Q}{M_1 \sqrt{2}} n_- - \frac{\lambda_1 M_1}{z_1 Q\sqrt{2}} n_+ + S_{1T} \approx \frac{\lambda_1}{M_1} P_1 + S_{1T}, \quad (25)$$

with for a pure state $\lambda_1^2 + S_{1T}^2 = 1$. In the approximations the $+$ components ($\propto 1/Q$) are neglected, as these are irrelevant compared to the $+$ components of the momenta in the hard part ($\propto Q$).

The Dirac structure of the quark–quark correlation function can be expanded in a number of amplitudes, i.e. functions of invariants built up from the quark and hadron momenta, constrained by hermiticity and parity [8,12]. Here we directly integrate the correlation function over k^+ , which up to order $1/Q$ can be parametrized as follows:

$$\begin{aligned} \frac{1}{4z} \int dk^+ \Delta(P_1, S_1; k) \Big|_{k^- = P_1^-/z, k_T} &= \frac{M_1}{4P_1^-} \left\{ E1 + D_1 \frac{\not{k}_1}{M_1} + D_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu P_1^\nu k_T^\rho S_{1T}^\sigma}{M_1^2} \right. \\ &+ D^\perp \frac{k_T}{M_1} + D_T \epsilon_{\mu\nu\rho\sigma} n_+^\mu n_-^\nu \gamma^\rho S_{1T}^\sigma + \lambda D_L^\perp \frac{\epsilon_{\mu\nu\rho\sigma} n_+^\mu n_-^\nu \gamma^\rho k_T^\sigma}{M_1} - E_s i\gamma_5 - G_{1s} \frac{\not{k}_1 \gamma_5}{M_1} \\ &- G_T' \frac{\not{k}_1 \gamma_5}{M_1} - G_s' \frac{k_T \gamma_5}{M_1} - H_{1T} \frac{i\sigma_{\mu\nu} \gamma_5 S_{1T}^\mu P_1^\nu}{M_1} - H_{1s}^\perp \frac{i\sigma_{\mu\nu} \gamma_5 k_T^\mu P_1^\nu}{M_1^2} \\ &\left. - H_T^\perp \frac{i\sigma_{\mu\nu} \gamma_5 S_{1T}^\mu k_T^\nu}{M_1} - H_s i\sigma_{\mu\nu} \gamma_5 n_-^\mu n_+^\nu + H_1^\perp \frac{\sigma_{\mu\nu} k_T^\mu P_1^\nu}{M_1^2} + H\sigma_{\mu\nu} n_-^\mu n_+^\nu \right\}, \quad (26) \end{aligned}$$

where the shorthand notation G_{1s} stands for the combination

$$G_{1s}(z, k_T) = \lambda_1 G_{1L} + G_{1T} \frac{(k_T \cdot S_{1T})}{M_1}, \quad (27)$$

etc. The functions E, D_1, \dots in Eq. (26) and G_{1L}, G_{1T}, \dots in G_{1s}, \dots are fragmentation functions. One wants to express the fragmentation functions in terms of the hadron momentum, hence, the arguments of the fragmentation functions are chosen to be the light-cone (momentum) fraction $z = P_1^-/k^-$ of the produced hadron with respect to the fragmenting quark and $k_T' \equiv -z k_T$, which is the transverse momentum of the hadron in a frame where the quark has no transverse momentum. In order to switch from quark to hadron transverse momentum a Lorentz transformation leaving k^- and P_1^- unchanged needs to be performed. The fragmentation functions are real and in fact, depend on z and $k_T'^2$ only.

Inverting the above expression, the fragmentation functions appear in specific Dirac projections of the correlation functions, integrated over k^+ ,

$$\begin{aligned} \Delta^{[I]}(z, k_T) &\equiv \frac{1}{4z} \int dk^+ \text{Tr}(\Delta \Gamma) \Big|_{k^- = P_1^-/z, k_T} \\ &= \sum_X \int \frac{dx^+ d^2 x_T}{4z (2\pi)^3} e^{ik \cdot x} \text{Tr}(0|\psi(x)|P_1, S_1; X) \langle P_1, S_1; X | \bar{\psi}(0) \Gamma | 0 \rangle \Big|_{x^-=0}, \quad (28) \end{aligned}$$

for which we can distinguish the leading fragmentation functions

$$\Delta^{[\gamma^-]}(z, \mathbf{k}_T) = D_1(z, \mathbf{k}_T'^2) + \frac{\epsilon_T^{ij} k_{Ti} S_{1Tj}}{M_1} D_{1T}^\perp(z, \mathbf{k}_T'^2), \quad (29)$$

$$\Delta^{[\gamma^- \gamma_5]}(z, \mathbf{k}_T) = G_{1s}(z, \mathbf{k}_T), \quad (30)$$

$$\Delta^{[i\sigma^{j-} \gamma_5]}(z, \mathbf{k}_T) = S_{1T}^i H_{1T}(z, \mathbf{k}_T'^2) + \frac{k_T^i}{M_1} H_{1s}^\perp(z, \mathbf{k}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1} H_1^\perp(z, \mathbf{k}_T'^2). \quad (31)$$

Furthermore we obtain the subleading projections (i, j are transverse indices)

$$\Delta^{[1]}(z, \mathbf{k}_T) = \frac{M_1}{P_1^-} E(z, \mathbf{k}_T'^2), \quad (32)$$

$$\begin{aligned} \Delta^{[\gamma']} (z, \mathbf{k}_T) &= \frac{k_T^i}{P_1^-} D^\perp(z, \mathbf{k}_T'^2) + \frac{\lambda_1 \epsilon_T^{ij} k_{Tj}}{P_1^-} D_L^\perp(z, \mathbf{k}_T'^2) \\ &\quad + \frac{M_1 \epsilon_T^{ij} S_{1Tj}}{P_1^-} D_T(z, \mathbf{k}_T'^2), \end{aligned} \quad (33)$$

$$\Delta^{[i\gamma_5]}(z, \mathbf{k}_T) = \frac{M_1}{P_1^-} E_s(z, \mathbf{k}_T), \quad (34)$$

$$\Delta^{[\gamma' \gamma_5]}(z, \mathbf{k}_T) = \frac{M_1 S_{1T}^i}{P_1^-} G_T'(z, \mathbf{k}_T'^2) + \frac{k_T^i}{P_1^-} G_s^\perp(z, \mathbf{k}_T), \quad (35)$$

$$\Delta^{[i\sigma^{jj} \gamma_5]}(z, \mathbf{k}_T) = \frac{S_{1T}^i k_T^j - k_T^i S_{1T}^j}{P_1^-} H_T^\perp(z, \mathbf{k}_T'^2) + \frac{M_1 \epsilon_T^{ij}}{P_1^-} H(z, \mathbf{k}_T'^2), \quad (36)$$

$$\Delta^{[i\sigma^{-+} \gamma_5]}(z, \mathbf{k}_T) = \frac{M_1}{P_1^-} H_s(z, \mathbf{k}_T). \quad (37)$$

We identified leading and subleading functions, which in principle start contributing at order 1 and $1/Q$, respectively. The order at which a function first can contribute depends on the power of M_1/P_1^- in front of the function as it appears in the projections. Each factor M_1/P_1^- leads to a suppression with a power of M_1/Q in cross sections. We will refer to the function multiplying a power $(M_1/P_1^-)^{t-2}$ as being of “twist” t . We note that this notion of twist, in analogy to the k_T -integrated case [31], is related but not equal to the one used for local operators in the OPE.

The naming scheme is as follows. All functions obtained after tracing with a scalar (1) or pseudoscalar ($i\gamma_5$) Dirac matrix are given the name $E_{...}$, those traced with a vector matrix (γ^μ) are given the name $D_{...}$, those traced with an axial vector matrix ($\gamma^\mu \gamma_5$) are given the name $G_{...}$ and, finally, those traced with the second rank tensor ($i\sigma^{\mu\nu} \gamma_5$) are given the name $H_{...}$. A subscript 1 is given to the leading functions, subscripts L or T refer to the connection with the hadron spin being longitudinal or transverse and a superscript \perp signals the explicit presence of transverse momenta with a non-contracted index. In the literature sometimes the fragmentation functions are denoted by lower-case names, but supplemented by a hat ($\hat{e}, \hat{g}, \hat{h}$), with the one exception that D is named \hat{f} . We note that after integration over \mathbf{k}_T several functions

disappear. In the case of $\Delta^{[i\sigma'\gamma_5]}$ and $\Delta^{[l'\gamma_5]}$ specific combinations remain, namely $H_1 \equiv H_{1T} + (k_T^2/2M_1^2)H_{1T}^\perp$ and $G_T \equiv G_T' + (k_T^2/2M_1^2)G_T^\perp$, respectively.

The choice of factors in the definition of the fragmentation functions is such that $\int dz d^2k_T' D_1(z, k_T') = N_h$, where N_h is the number of produced hadrons. The twist-two fragmentation functions have natural interpretations as decay functions. The projection $\Delta^{[l'\gamma^-]}$ is (after proper normalizing) the probability of a quark to produce a spin- $\frac{1}{2}$ hadron in a specific spin state, $\Delta^{[l'\gamma^-\gamma_5]}$ is the difference of the probabilities for a chirally right and chirally left quark to produce such a hadron, while $\Delta^{[i\sigma'\gamma_5]}$ is the difference of opposite transverse spin states (along direction i) of a quark to produce such a hadron.

Note that the decay probability for an unpolarized quark with non-zero transverse momentum can lead to a transverse polarization in the production of spin- $\frac{1}{2}$ particles. This polarization is orthogonal to the quark transverse momentum and the probability is given by the function D_{1T}^\perp . In the same way, oppositely transversely polarized quarks with non-zero transverse momentum can produce unpolarized hadrons or spinless particles, with different probabilities. This difference is described by the function H_1^\perp , which is the one appearing in the so-called Collins effect [14], which shows up as a single transverse spin asymmetry in semi-inclusive DIS, and arises due to intrinsic transverse momentum.

The functions D_{1T}^\perp and H_1^\perp are examples of what are generally called “time-reversal-odd” functions. This somewhat misleading terminology refers to the behavior of the functions under the so-called *naive* time-reversal operation T_N [32], which acts as follows on the correlation functions:

$$\Delta(P_1, S_1; k) \xrightarrow{T_N} (\gamma_5 C \Delta(\bar{P}_1, \bar{S}_1; \bar{k}) C^\dagger \gamma_5)^*, \quad (38)$$

where $\bar{k} = (k^0, -\mathbf{k})$, etc. If T_N invariance would apply, the functions D_{1T}^\perp , H_1^\perp , D_L^\perp , D_T , E_L , E_T and H would be purely imaginary. On the other hand, hermiticity requires the functions to be real, so these functions should then vanish.

The operation T_N differs from the actual time-reversal operation T in that the former does not transform *in* into *out* states and vice versa. Due to final state interactions, the *out* state $|P_1, S_1; X\rangle$ in $\Delta(P_1, S_1; k)$ is not a plane wave state and thus, is not simply related to an *in* state. Therefore, one has $T_N \neq T$ and since T itself does not pose any constraints on the functions, they need not vanish.

In the analogous case of distribution functions, which are derived from matrix elements with plane wave states, $T = T_N$ and therefore there are no “time-reversal-odd” distribution functions.

The H_{\dots} and E functions (but not E_s) are called chiral-odd functions because they are non-diagonal in the chirality basis, so they arise either accompanied by a quark mass term or by another chiral-odd function, such that the product is again chiral-even [33,9].

The quark–gluon–quark correlation functions can be expressed in terms of the quark–quark correlation functions with help of the classical equations of motion (e.o.m.). These can be used inside hadronic matrix elements [4]. If we again define Dirac projections,

$$\begin{aligned}
\Delta_A^{\alpha[\Gamma]}(z, \mathbf{k}_T) &= \frac{1}{4z} \int dk^+ d^4 k_1 \operatorname{Tr}(\Delta_A^\alpha \Gamma) \Big|_{k^- = P_1^- / z, \mathbf{k}_T} \\
&= \sum_X \int \frac{dx^+ d^2 x_\perp}{4z (2\pi)^3} e^{ik \cdot x} \\
&\quad \times \operatorname{Tr} \langle 0 | \psi(x) g A_T^\alpha(x) | P_1, S_1; X \rangle \langle P_1, S_1; X | \bar{\psi}(0) \Gamma | 0 \rangle \Big|_{x^- = 0}, \quad (39)
\end{aligned}$$

we find as a consequence of the e.o.m.

$$\begin{aligned}
\Delta_{A\alpha}^{[\sigma^{\alpha-}]} &= -\epsilon_T^{\alpha\beta} \Delta_A^{\alpha[i\sigma^{\beta-}\gamma_s]} \\
&= \frac{M_1}{z} (\tilde{H} + i\tilde{E}) - \epsilon_T^{ij} k_{Ti} S_{1Tj} \left(\frac{1}{z} \tilde{H}_T^\perp + i \frac{m}{M_1} D_{1T}^\perp \right), \quad (40)
\end{aligned}$$

$$\Delta_{A\alpha}^{[i\sigma^{\alpha-}\gamma_s]} = \frac{M_1}{z} (\tilde{H}_s + i\tilde{E}_s), \quad (41)$$

$$\begin{aligned}
\Delta_A^{\alpha[\gamma^-]} + i\epsilon_T^{\alpha\beta} \Delta_{A\beta}^{[\gamma^-\gamma_s]} &= k_T^\alpha \left(\frac{1}{z} \tilde{D}^\perp + i \frac{m}{M_1} H_1^\perp \right) - \frac{(k_T^\alpha k_T^i + \frac{1}{2} k_T^2 g_T^{\alpha i})}{M_1} \epsilon_T^{ij} S_{1Tj} D_{1T}^\perp \\
&\quad + i\epsilon_T^{\alpha\beta} k_{T\beta} \frac{1}{z} (\tilde{G}_s^\perp - i\lambda_1 \tilde{D}_L^\perp) + i\epsilon_T^{\alpha\beta} S_{1T\beta} \frac{M_1}{z} (\tilde{G}_T' - i\tilde{D}_T'), \quad (42)
\end{aligned}$$

where the functions indicated with a tilde (\tilde{H} , \tilde{E} , ...) differ from the corresponding twist-three functions (H , E , ...) by a twist-two part, namely

$$E = \frac{m}{M_1} z D_1 + \tilde{E}, \quad (43)$$

$$D^\perp = z D_1 + \tilde{D}^\perp, \quad (44)$$

$$D_L^\perp = \tilde{D}_L^\perp, \quad (45)$$

$$D_T = -\frac{k_T^2}{2M_1^2} z D_{1T}^\perp + \tilde{D}_T, \quad (46)$$

$$E_s = \tilde{E}_s, \quad (47)$$

$$G_T' = \frac{m}{M_1} z H_{1T} + \tilde{G}_T', \quad (48)$$

$$G_s^\perp = z G_{1s} + \frac{m}{M_1} z H_{1s}^\perp + \tilde{G}_s^\perp, \quad (49)$$

$$G_T = \frac{k_T^2}{2M_1^2} z G_{1T} + \frac{m}{M_1} z H_1 + \tilde{G}_T, \quad (50)$$

$$H_T^\perp = z H_{1T} + \tilde{H}_T^\perp, \quad (51)$$

$$H = -\frac{k_T^2}{M_1^2} z H_1^\perp + \tilde{H}, \quad (52)$$

$$H_s = \frac{m}{M_1} z G_{1s} - \frac{k_T \cdot S_{1T}}{M_1} z H_{1T} - \frac{k_T^2}{M_1^2} z H_{1s}^\perp + \tilde{H}_s. \quad (53)$$

We have included G_T in this list since it is relevant for k_T' -integrated functions and note that $\tilde{G}_T = \tilde{G}_T' + (k_T^2/2M_1^2)\tilde{G}_T^\perp$. The functions in Eqs. (40) to (42) are interaction dependent and vanish for the case of a quark fragmenting in a quark (as can be checked with the help of Appendix A). Note that the time-reversal-odd twist-two functions¹ D_{17}^\perp and H_1^\perp are in fact interaction dependent. Their presence is due to final state interactions of the produced hadrons, which after all are strong interactions. The separation of twist-three functions in this way is analogous to the case of the distribution function $g_T = g_1 + g_2$, and the twist-two parts could be called Wandzura–Wilczek parts [34].

For the fragmentation of an antiquark most things are analogous to the quark fragmentation. The major difference in our case is that the role of the + and – direction is reversed. We will denote the antiquark correlation functions by $\bar{\Delta}(P_2, S_2; p)$. These should be defined consistently with the replacement $\psi \rightarrow \psi^c = C\bar{\psi}^T$, or $\bar{\Delta}^{[\Gamma]} = \Delta^{c[\Gamma]}$ for $\Gamma = \gamma_\mu, i\sigma_{\mu\nu}\gamma_5, i\gamma_5$ and $\bar{\Delta}^{[\Gamma]} = -\Delta^{c[\Gamma]}$ for $\Gamma = \mathbf{1}, \gamma_\mu\gamma_5$, where we have defined the projections as

$$\bar{\Delta}^{[\Gamma]}(\bar{z}, \mathbf{p}_T) = \frac{1}{4\bar{z}} \int dp^- \text{Tr}(\bar{\Delta}\Gamma) \Big|_{p^+ = P_2^+/\bar{z}, \mathbf{p}_T}, \quad (54)$$

where we make the following Sudakov decomposition for the antiquark momentum p :

$$p \equiv \frac{z_2 Q}{\bar{z}\sqrt{2}} n_+ + \frac{\bar{z}(p^2 + \mathbf{p}_T^2)}{z_2 Q \sqrt{2}} n_- + p_T \approx \frac{1}{\bar{z}} P_2 + p_T. \quad (55)$$

Similarly, we decompose the spin vector S_2 ,

$$S_2 \equiv \frac{\lambda_2 z_2 Q}{M_2 \sqrt{2}} n_+ - \frac{\lambda_2 M_2}{z_2 Q \sqrt{2}} n_- + S_{2T} \approx \frac{\lambda_2}{M_2} P_2 + S_{2T}, \quad (56)$$

with for a pure state $\lambda_2^2 + S_{2T}^2 = 1$. The antiquark fragmentation functions are denoted by $\bar{D}_1(\bar{z}, \mathbf{p}_T'^2), \dots$, with $\mathbf{p}_T' = -\bar{z}\mathbf{p}_T$, in full analogy to the quark fragmentation functions. The antiquark fragmentation functions are obtained from

$$\bar{\Delta}_{ij}(P_2, S_2; p) = \sum_x \frac{1}{(2\pi)^4} \int d^4x e^{-ip \cdot x} \langle 0 | \bar{\psi}_j(0) | P_2, S_2; X \rangle \langle P_2, S_2; X | \psi_i(x) | 0 \rangle. \quad (57)$$

Although the antiquark–gluon–antiquark correlation functions are straightforwardly defined, we still give here the relations which follow from the e.o.m., since these differ non-trivially from those for the quark–gluon–quark correlation functions. We will not use tilde functions here, since in the non-symmetric frame in which we will express the hadron tensor (non-symmetric between quark and antiquark fragmentation part), they do not show up in a natural way.

¹ The arbitrariness in the definition of \tilde{D}_T and \tilde{H} in Eqs. (46) and (52) is fixed by the requirement that the functions D_{17}^\perp and H_1^\perp do not appear in the integrated versions of Eqs. (40)–(42).

$$\begin{aligned} \overline{\Delta}_{A\alpha}^{[\sigma^{\alpha+}]} &= \epsilon_T^{\alpha\beta} \overline{\Delta}_A^{[\sigma^{\beta+} \gamma_5]} = i \left(\frac{M_2}{\bar{z}} \bar{E} - m \bar{D}_1 + i \frac{M_2}{\bar{z}} \bar{H} - i \frac{p_T^2}{M_2} \bar{H}_1^\perp \right) \\ &\quad + \epsilon_T^{ij} p_{Ti} S_{2Tj} \left(\bar{H}_{1T} - \frac{1}{\bar{z}} \bar{H}_T^\perp + i \frac{m}{M_2} \bar{D}_{1T}^\perp \right), \end{aligned} \quad (58)$$

$$\overline{\Delta}_{A\alpha}^{[i\sigma^{\alpha+} \gamma_5]} = -\frac{M_2}{\bar{z}} \bar{H}_s + m \bar{G}_{1s} + i \frac{M_2}{\bar{z}} \bar{E}_s + (p_T \cdot S_{2T}) \bar{H}_{1T} + \frac{p_T^2}{M_2} \bar{H}_{1s}^\perp, \quad (59)$$

$$\begin{aligned} \overline{\Delta}_A^{\alpha[\gamma^+]} - i \epsilon_T^{\alpha\beta} \overline{\Delta}_{A\beta}^{[\gamma^+ \gamma_5]} &= -p_T^\alpha \left(\frac{1}{\bar{z}} \bar{D}^\perp - \bar{D}_1 - i \frac{m}{M_2} \bar{H}_1^\perp \right) - \frac{p_T^\alpha}{M_2} \epsilon_T^{ij} p_{Ti} S_{2Tj} \bar{D}_{1T}^\perp \\ &\quad - i \epsilon_T^{\alpha\beta} p_{T\beta} \left(\frac{1}{\bar{z}} \bar{G}_s^\perp - \bar{G}_{1s} - \frac{m}{M_2} \bar{H}_{1s}^\perp + i \frac{\lambda_2}{\bar{z}} \bar{D}_L^\perp \right) \\ &\quad - i \epsilon_T^{\alpha\beta} S_{2T\beta} \left(\frac{M_2}{\bar{z}} \bar{G}_T' - m \bar{H}_{1T} + i \frac{M_2}{\bar{z}} \bar{D}_T \right). \end{aligned} \quad (60)$$

Until now we have not commented on color gauge invariance of the correlation functions. As given above they are gauge-invariant quantities displayed in a specific gauge. In general, one has to include path-ordered exponentials, in order to compensate for the gauge non-invariance due to the non-locality of the operators. Such a link operator is of the form

$$\mathcal{L}(0, x) = \mathcal{P} \exp \left(-ig \int_0^x dz^\mu A_\mu(z) \right). \quad (61)$$

At this point we assume that matrix elements with multiple A^- -gluon fields in $\Delta_A^-, \Delta_{AA}^{--}, \dots$ (multiple A^+ -gluon fields in $\overline{\Delta}_A^+, \overline{\Delta}_{AA}^{++}, \dots$) will combine into an appropriate link operator with its path along the $+$ direction ($-$ direction) in Δ ($\overline{\Delta}$) (cf. Ref. [10]). For the k_T -dependent functions which involve transverse separations, the path from the point 0 to x in Δ will run along the $+$ direction via $x^+ = \infty$. The transverse part of the path, which is at ∞ , does not contribute, since matrix elements are assumed to vanish there.

There remains one issue to be addressed, namely the explicit A_T in Δ_A^α is not gauge invariant. Nevertheless, using the covariant derivative, one can express this A_T in terms of D_T and ∂_T , such that

$$\Delta_A^{\alpha[F]}(z, k_T) = \Delta_D^{\alpha[F]}(z, k_T) - k^\alpha \Delta^{[F]}(z, k_T), \quad (62)$$

where

$$\begin{aligned} \sum_x \frac{1}{(2\pi)^4} \int d^4x e^{ik \cdot x} \langle 0 | \psi_i(x) i \partial^\mu | P_1, S_1; X \rangle \langle P_1, S_1; X | \bar{\psi}_j(0) | 0 \rangle \\ = k^\mu \Delta_{ij}(P_1, S_1; k). \end{aligned} \quad (63)$$

Hence, we see that (again after inclusion of a link operator, assumed to arise from $\Delta_{AA}^{\alpha-}, \Delta_{AAA}^{\alpha--}, \dots$) Δ_A^α is a color gauge-invariant quantity and therefore, so are the interaction-dependent parts of the twist-three fragmentation functions.

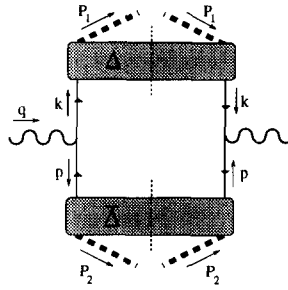


Fig. 3. Quark diagram contributing to e^+e^- annihilation in leading order. There is a similar diagram with reversed fermion flow.

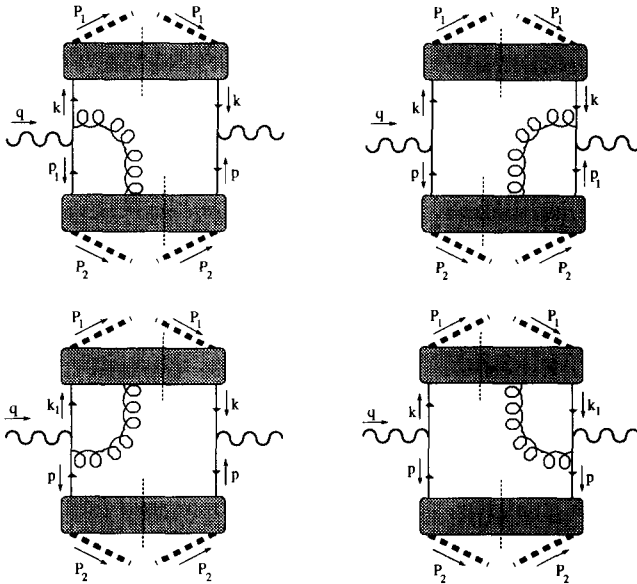


Fig. 4. Diagrams contributing to e^+e^- annihilation at order $1/Q$.

4. The complete tree-level calculation

Up to order $1/Q$ there are five tree-level diagrams to consider. The simplest diagram (Fig. 3) involving only quarks contributes at order 1 and $1/Q$, the other four (Fig. 4) involve one gluon which connects to one of the two soft parts. Note that one power of the coupling constant is included in the definition of the soft part, such that the diagrams are of order $(\alpha_s)^0$.

The momentum conserving delta function at the photon vertex is written as (neglecting $1/Q^2$ contributions)

$$\delta^4(q - k - p) = \delta(q^+ - p^+) \delta(q^- - k^-) \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T), \quad (64)$$

fixing $P_2^+/\bar{z} = p^+ = q^+ = P_2^+/z_2$ and $P_1^-/z = k^- = q^- = P_1^-/z_1$. Eq. (64) shows why

only the k^+ and p^- -integrated correlation functions are relevant. Note that the quark transverse momentum integrations are linked. The five diagrams lead to the following expression for the full result up to order $1/Q$:

$$\begin{aligned} \mathcal{W}^{\mu\nu} = & 3e^2 \int dp^- dk^+ d^2 p_T d^2 k_T \delta^2(p_T + k_T - q_T) \left\{ \text{Tr} \left(\bar{\Delta}(p) \gamma^\mu \Delta(k) \gamma^\nu \right) \right. \\ & - \text{Tr} \left(\bar{\Delta}_A^\alpha(p) \gamma^\mu \Delta(k) \gamma_\alpha \frac{\not{p}_+}{Q\sqrt{2}} \gamma^\nu \right) \\ & - \text{Tr} \left(\left(\gamma_0 \bar{\Delta}_A^{\alpha\dagger}(p) \gamma_0 \right) \gamma^\mu \frac{\not{p}_+}{Q\sqrt{2}} \gamma_\alpha \Delta(k) \gamma^\nu \right) \\ & + \text{Tr} \left(\bar{\Delta}(p) \gamma^\mu \left(\gamma_0 \Delta_A^{\alpha\dagger}(k) \gamma_0 \right) \gamma^\nu \frac{\not{p}_-}{Q\sqrt{2}} \gamma_\alpha \right) \\ & \left. + \text{Tr} \left(\bar{\Delta}(p) \gamma_\alpha \frac{\not{p}_-}{Q\sqrt{2}} \gamma^\mu \Delta_A^\alpha(k) \gamma^\nu \right) \right\} \Big|_{p^+ k^-}. \end{aligned} \quad (65)$$

The factor 3 originates from the color summation. We have omitted the flavor indices and summation; furthermore, there is a contribution from diagrams with reversed fermion flow, which results from the above expression by replacing $\mu \leftrightarrow \nu$ and $q \rightarrow -q$.

In the expression the terms with \not{p}_\pm arise from the fermion propagators in the hard part neglecting contributions that will appear suppressed by powers of Q^2 ,

$$\begin{aligned} \frac{\not{q} - \not{p}_1 + m}{(q - p_1)^2 - m^2} &\approx \frac{(q^+ - p_1^+) \gamma^-}{2(q^+ - p_1^+) q^-} = \frac{\gamma^-}{2q^-} = \frac{\not{p}_+}{Q\sqrt{2}}, \\ \frac{\not{k}_1 - \not{q} + m}{(k_1 - q)^2 - m^2} &\approx \frac{(k_1^- - q^-) \gamma^+}{-2(k_1^- - q^-) q^+} = \frac{\gamma^+}{-2q^+} = -\frac{\not{p}_-}{Q\sqrt{2}}, \end{aligned} \quad (66)$$

where the approximate sign holds true only when the propagators are embedded in the diagrams. The quantity $\Delta_A^\alpha(k)$ arises from integrating out the second argument of $\Delta_A^\alpha(k, k_1)$ instead of the first which yields the combination $\gamma_0 \Delta_A^{\alpha\dagger}(k) \gamma_0$,

$$\begin{aligned} & \int \frac{d^4 k_1}{(2\pi)^4} \Delta_{Aij}^\alpha(P_1, S_1; k, k_1) \\ &= \sum_x \frac{1}{(2\pi)^4} \int d^4 x e^{ik \cdot x} \langle 0 | \psi_i(x) g A_T^\alpha(x) | P_1, S_1; X \rangle \langle P_1, S_1; X | \bar{\psi}_j(0) | 0 \rangle \\ &= \Delta_{Aij}^\alpha(P_1, S_1; k), \end{aligned} \quad (67)$$

$$\begin{aligned} & \int \frac{d^4 k_1}{(2\pi)^4} \Delta_{Aij}^\alpha(P_1, S_1; k_1, k) \\ &= \sum_x \frac{1}{(2\pi)^4} \int d^4 x e^{ik \cdot x} \langle 0 | \psi_i(x) | P_1, S_1; X \rangle \langle P_1, S_1; X | g A_T^\alpha(0) \bar{\psi}_j(0) | 0 \rangle \\ &= (\gamma_0 \Delta_A^{\alpha\dagger} \gamma_0)_{ij}(P_1, S_1; k) \end{aligned} \quad (68)$$

and similarly for $\bar{\Delta}_A^\alpha(p)$ and $\gamma_0 \bar{\Delta}_A^{\alpha\dagger}(p) \gamma_0$. To deal with these combinations one can use the relation

$$\left(\gamma_0 \Delta_A^{\alpha\dagger} \gamma_0\right)^{[I]} = \left(\Delta_A^{\alpha[I]}\right)^* \quad (69)$$

and a similar one for $\gamma_0 \bar{\Delta}_A^{\alpha\dagger}(p) \gamma_0$.

To obtain the expressions for the symmetric and antisymmetric parts of the hadron tensor we expand all vectors in Δ , $\bar{\Delta}$, Δ_A^α and $\bar{\Delta}_A^\alpha$ in the perpendicular basis (\hat{t} , \hat{z} and \perp directions). In particular, we re-express the transverse vectors k_T , p_T , S_{1T} and S_{2T} in terms of their perpendicular parts and a part along \hat{t} and \hat{z} . For this we need

$$g_T^{\mu\nu} = g_\perp^{\mu\rho} g_{T\rho}^\nu - \frac{Q_T}{Q} (\hat{t}^\mu + \hat{z}^\mu) \hat{h}^\nu. \quad (70)$$

We will refer to the perpendicular projections as k_\perp , etc. (instead of the fully logical name, which would be $k_{T\perp}$). Thus

$$k_\perp^\mu \equiv g_\perp^{\mu\nu} k_{T\nu} = k_T^\mu + \frac{q_T \cdot k_T}{Q} (\hat{t}^\mu + \hat{z}^\mu), \quad (71)$$

and similarly for p_\perp , $S_{1\perp}$ and $S_{2\perp}$. We note that for these four vectors with this definition the two-component perpendicular parts are the same as the two-component transverse parts, i.e. $k_\perp = k_T$, $S_{1\perp} = S_{1T}$, etc.

The full expressions for the symmetric and antisymmetric parts of the hadron tensor (expressed in the perpendicular frame defined in Section 2) are given in Appendix B. We note that the expressions are not symmetric in the interchange of the hadrons 1 and 2, because the choice of perpendicular direction ($P_{2\perp} \equiv 0$) is non-symmetric.

The cross sections are obtained from the hadron tensor after contraction with the lepton tensor,

$$\begin{aligned} L^{\mu\nu} = Q^2 & \left[- (1 - 2y + 2y^2) g_\perp^{\mu\nu} + 4y(1 - y) \hat{z}^\mu \hat{z}^\nu \right. \\ & - 4y(1 - y) \left(\hat{\ell}^\mu \hat{\ell}^\nu + \frac{1}{2} g_\perp^{\mu\nu} \right) - 2(1 - 2y) \sqrt{y(1 - y)} \hat{z}^{\{\mu} \hat{\ell}^{\nu\}} \\ & \left. + i\lambda_e(1 - 2y) \epsilon_\perp^{\mu\nu} - 2i\lambda_e \sqrt{y(1 - y)} \hat{\ell}_\rho \epsilon_\perp^{\rho\{\mu} \hat{z}^{\nu\}} \right], \quad (72) \end{aligned}$$

where $\{\mu\nu\}$ indicates symmetrization of indices and $[\mu\nu]$ indicates antisymmetrization. The fraction y is defined to be $y = P_2 \cdot l / P_2 \cdot q \approx l^- / q^-$, which in the lepton center of mass frame equals $y = (1 \pm \cos \theta_2) / 2$, where θ_2 is the angle of hadron 2 with respect to the momentum of the incoming leptons. The contractions of specific tensor structures in the hadron tensor, given in Table 1, contain azimuthal angles inside the perpendicular plane defined with respect to \hat{l}_\perp^μ , defined to be the normalized perpendicular part of the lepton momentum l , $\hat{l}_\perp^\mu = l_\perp^\mu / (Q\sqrt{y(1 - y)})$,

$$\hat{l}_\perp \cdot a_\perp = -|a_\perp| \cos \phi_a, \quad (73)$$

$$\epsilon_\perp^{\mu\nu} \hat{l}_\perp^\mu a_{\perp\nu} = |a_\perp| \sin \phi_a. \quad (74)$$

Table 1

Contractions of the lepton tensor $L_{\mu\nu}$ with tensor structures appearing in the hadron tensor

$w^{\mu\nu}$	$L_{\mu\nu} w^{\mu\nu} / (4Q^2)$
$-g_{\perp}^{\mu\nu}$	$(\frac{1}{2} - y + y^2)$
$a_{\perp}^{\{\mu} b_{\perp}^{\nu\}} - (a_{\perp} \cdot b_{\perp}) g_{\perp}^{\mu\nu}$	$-y(1-y) a_{\perp} b_{\perp} \cos(\phi_a + \phi_b)$
$\frac{1}{2} \left(a_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} b_{\perp\rho} + b_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} a_{\perp\rho} \right)$ $= a_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} b_{\perp\rho} - (\epsilon_{\perp}^{\rho\sigma} a_{\perp\rho} b_{\perp\sigma}) g_{\perp}^{\mu\nu}$	$y(1-y) a_{\perp} b_{\perp} \sin(\phi_a + \phi_b)$
$\hat{z}^{\{\mu} a_{\perp}^{\nu\}}$	$-(1-2y) \sqrt{y(1-y)} a_{\perp} \cos \phi_a$
$\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} a_{\perp\rho}$	$(1-2y) \sqrt{y(1-y)} a_{\perp} \sin \phi_a$
$i \epsilon_{\perp}^{\mu\nu}$	$-\lambda_e \left(\frac{1}{2} - y \right)$
$i a_{\perp}^{\{\mu} b_{\perp}^{\nu\}}$	$-\lambda_e \left(\frac{1}{2} - y \right) a_{\perp} b_{\perp} \sin(\phi_b - \phi_a)$
$i \hat{z}^{\{\mu} a_{\perp}^{\nu\}}$	$\lambda_e \sqrt{y(1-y)} a_{\perp} \sin \phi_a$
$i \hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} a_{\perp\rho}$	$\lambda_e \sqrt{y(1-y)} a_{\perp} \cos \phi_a$

5. Integration over transverse photon momentum

In the next sections we will discuss explicit expressions for cross sections. Instead of giving the complete cross section, which can be obtained from the hadron tensor (Appendix B), we treat a number of special cases. In this section we consider cross sections integrated over all transverse momenta.

After integration over the transverse momentum of the photon (or equivalently over the perpendicular momentum of hadron one $\mathbf{P}_{1\perp} = -z_1 \mathbf{q}_T$), the integrations over \mathbf{k}_T and \mathbf{p}_T in the hadron tensor (Eqs. (B.1) and (B.2)) can be performed leading to

$$\begin{aligned}
 \int d^2 \mathbf{q}_T \mathcal{W}_S^{\mu\nu} = & 12e^2 z_1 z_2 \sum_{a,\bar{a}} e_a^2 \left\{ -g_{\perp}^{\mu\nu} \left[D_1 \bar{D}_1 - \lambda_1 \lambda_2 G_1 \bar{G}_1 \right] \right. \\
 & - \left(S_{1\perp}^{\{\mu} S_{2\perp}^{\nu\}} + g_{\perp}^{\mu\nu} S_{1\perp} \cdot S_{2\perp} \right) \left[H_1 \bar{H}_1 \right] \\
 & - 2 \frac{\hat{z}^{\{\mu} S_{1\perp}^{\nu\}}}{Q} \lambda_2 \left[M_1 \frac{\tilde{G}_T}{z_1} \bar{G}_1 + M_2 H_1 \frac{\bar{H}_L}{z_2} \right] \\
 & + 2 \frac{\hat{z}^{\{\mu} S_{2\perp}^{\nu\}}}{Q} \lambda_1 \left[M_2 G_1 \frac{\bar{G}_T}{z_2} + M_1 \frac{\tilde{H}_L}{z_1} \bar{H}_1 \right] \\
 & + 2 \frac{\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} S_{1\perp\rho}}{Q} \left[M_1 \frac{\tilde{D}_T}{z_1} \bar{D}_1 + M_2 H_1 \frac{\bar{H}}{z_2} \right] \\
 & \left. + 2 \frac{\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} S_{2\perp\rho}}{Q} \left[M_2 D_1 \frac{\bar{D}_T}{z_2} + M_1 \frac{\tilde{H}}{z_1} \bar{H}_1 \right] \right\} \quad (75)
 \end{aligned}$$

and

$$\begin{aligned}
\int d^2 q_T \mathcal{W}_A^{\mu\nu} = & 12e^2 z_1 z_2 \sum_{a,\bar{a}} e_a^2 \left\{ i\epsilon_{\perp}^{\mu\nu} \left[\lambda_1 G_1 \bar{D}_1 - \lambda_2 D_1 \bar{G}_1 \right] \right. \\
& + 2i \frac{\hat{z}^{[\mu} S_{1\perp}^{\nu]}}{Q} \lambda_2 \left[M_1 \frac{\tilde{D}_T}{z_1} \bar{G}_1 - M_2 H_1 \frac{\tilde{E}_L}{z_2} \right] \\
& + 2i \frac{\hat{z}^{[\mu} S_{2\perp}^{\nu]}}{Q} \lambda_1 \left[-M_2 G_1 \frac{\bar{D}_T}{z_2} + M_1 \frac{\tilde{E}_L}{z_1} \bar{H}_1 \right] \\
& + 2i \frac{\hat{z}^{[\mu} \epsilon_{\perp}^{\nu]\rho} S_{1\perp\rho}}{Q} \left[M_1 \frac{\tilde{G}_T}{z_1} \bar{D}_1 + M_2 H_1 \frac{\tilde{E}}{z_2} \right] \\
& \left. + 2i \frac{\hat{z}^{[\mu} \epsilon_{\perp}^{\nu]\rho} S_{2\perp\rho}}{Q} \left[M_2 D_1 \frac{\bar{G}_T}{z_2} + M_1 \frac{\tilde{E}}{z_1} \bar{H}_1 \right] \right\}. \quad (76)
\end{aligned}$$

We have now included the summation over flavor indices and e_a is the quark charge in units of e . The fragmentation functions are flavor dependent and only depend on the longitudinal momentum fractions, e.g. $D_1 \bar{D}_1 = D_1^a(z_1) \bar{D}_1^a(z_2)$. The result is expressed in terms of the fragmentation functions which survive the k_T integration of the Dirac projections of the correlation functions (cf. Eqs. (29) to (37)): $D_1, G_1 = G_{1L}, H_1, E, E_L, G_T, H, H_L, D_T$ [35,36].

Note that the tilde functions arise naturally in the quark fragmentation region. The reason that this does not occur for the antiquark fragmentation is due to the non-symmetric choice of frame. This non-symmetric feature only shows up at subleading order. The leading order is symmetric, since $\epsilon_{\perp}^{\mu\nu}$ acquires a minus sign, due to the interchange of the vectors n_+ and n_- .

From the hadron tensors we easily arrive at the following expressions for the cross sections, where we separate the cross sections into parts for unpolarized (O) and polarized (L) leptons:

$$\begin{aligned}
\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2} = & \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 \left\{ A(y) \left(D_1 \bar{D}_1 - \lambda_1 \lambda_2 G_1 \bar{G}_1 \right) \right. \\
& + B(y) |S_{1T}| |S_{2T}| \cos(\phi_{S_1} + \phi_{S_2}) \left(H_1 \bar{H}_1 \right) \\
& + C(y) D(y) |S_{1T}| \sin(\phi_{S_1}) \left(\frac{2M_1}{Q} \frac{\tilde{D}_T}{z_1} \bar{D}_1 + \frac{2M_2}{Q} H_1 \frac{\tilde{H}}{z_2} \right) \\
& + C(y) D(y) |S_{2T}| \sin(\phi_{S_2}) \left(\frac{2M_2}{Q} D_1 \frac{\bar{D}_T}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}}{z_1} \bar{H}_1 \right) \\
& + C(y) D(y) \lambda_2 |S_{1T}| \cos(\phi_{S_1}) \left(\frac{2M_1}{Q} \frac{\tilde{G}_T}{z_1} \bar{G}_1 + \frac{2M_2}{Q} H_1 \frac{\tilde{H}_L}{z_2} \right) \\
& \left. - C(y) D(y) \lambda_1 |S_{2T}| \cos(\phi_{S_2}) \left(\frac{2M_2}{Q} G_1 \frac{\bar{G}_T}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}_L}{z_1} \bar{H}_1 \right) \right\} \quad (77)
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\sigma^L(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2} = & \frac{3\alpha^2}{Q^2} \lambda_e \sum_{a,\bar{a}} e_a^2 \left\{ \frac{C(y)}{2} \left(\lambda_2 D_1 \bar{G}_1 - \lambda_1 G_1 \bar{D}_1 \right) \right. \\
& + D(y) |S_{2T}| \cos(\phi_{S_2}) \left(\frac{2M_2}{Q} D_1 \frac{\bar{G}_T}{z_2} + \frac{2M_1}{Q} \frac{\bar{E}}{z_1} \bar{H}_1 \right) \\
& + D(y) |S_{1T}| \cos(\phi_{S_1}) \left(\frac{2M_1}{Q} \frac{\bar{G}_T}{z_1} \bar{D}_1 + \frac{2M_2}{Q} H_1 \frac{\bar{E}}{z_2} \right) \\
& - D(y) \lambda_1 |S_{2T}| \sin(\phi_{S_2}) \left(\frac{2M_2}{Q} G_1 \frac{\bar{D}_T}{z_2} - \frac{2M_1}{Q} \frac{\bar{E}_L}{z_1} \bar{H}_1 \right) \\
& \left. + D(y) \lambda_2 |S_{1T}| \sin(\phi_{S_1}) \left(\frac{2M_1}{Q} \frac{\bar{D}_T}{z_1} \bar{G}_1 - \frac{2M_2}{Q} H_1 \frac{\bar{E}_L}{z_2} \right) \right\}, \quad (78)
\end{aligned}$$

where $d\Omega = 2dy d\phi^l$, with ϕ^l giving the orientation of \hat{l}_\perp^μ , see Fig. 1. Note that on the r.h.s. of the above equations the dependence on ϕ^l enters in the azimuthal angles, which are defined with respect to \hat{l}_\perp^μ , cf. Eqs. (73) and (74). We use the following factors:

$$\begin{aligned}
A(y) &= \left(\frac{1}{2} - y + y^2 \right), \\
B(y) &= y(1 - y), \\
C(y) &= 1 - 2y, \\
D(y) &= \sqrt{y(1 - y)}. \quad (79)
\end{aligned}$$

The first three terms in Eq. (77) coincide with the ones found in Ref. [37], if one neglects the contributions associated to Z exchange in their Eq. (45). One observes that besides these three leading contributions, one finds subleading single and double spin azimuthal asymmetries.

To reduce the expression to the one-particle inclusive cross section, one must take the fragmentation functions for a quark fragmenting into a quark (see Appendix A) and sum over spins. Only $D_1^q(z_1)$ survives and after summation over spins becomes a delta function. We find for the one-hadron inclusive integrated cross sections (using h as running index instead of 2 and realizing that $\bar{D}_1^q = D_1^q$)

$$\begin{aligned}
\frac{d\sigma^O(e^+e^- \rightarrow hX)}{d\Omega dz_h} = & \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 \left\{ A(y) D_1^q(z_h) \right. \\
& \left. + C(y) D(y) |S_{hT}| \sin(\phi_{S_h}) \frac{2M_h}{Q} \frac{D_T^q(z_h)}{z_h} \right\} \quad (80)
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\sigma^L(e^+e^- \rightarrow hX)}{d\Omega dz_h} = & \frac{3\alpha^2}{Q^2} \lambda_e \sum_{a,\bar{a}} e_a^2 \left\{ -\frac{C(y)}{2} \lambda_h G_1^q(z_h) \right. \\
& \left. + D(y) |S_{hT}| \cos(\phi_{S_h}) \frac{2M_h}{Q} \frac{G_T^q(z_h)}{z_h} \right\}. \quad (81)
\end{aligned}$$

If the hadrons are unpolarized we find

$$\frac{d\sigma^O(e^+e^- \rightarrow hX)}{d\Omega dz_h} = \frac{3\alpha^2}{Q^2} A(y) \sum_{a,\bar{a}} e_a^2 D_1^a(z_h), \quad (82)$$

$$\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2} = \frac{3\alpha^2}{Q^2} A(y) \sum_{a,\bar{a}} e_a^2 D_1^a(z_1) \bar{D}_1^a(z_2) \quad (83)$$

and $d\sigma^L = 0$ in both cases. Hence we find for the number of produced particles

$$N_h(z_h) = \sum_{a,\bar{a}} e_a^2 D_1^a(z_h) / \sum_{a,\bar{a}} e_a^2, \quad (84)$$

$$N_{h_1 h_2}(z_1, z_2) = \sum_{a,\bar{a}} e_a^2 D_1^a(z_1) \bar{D}_1^a(z_2) / \sum_{a,\bar{a}} e_a^2. \quad (85)$$

The case of $S_h = 0$ gives the number of particles produced per spin degree of freedom, while the part proportional to λ_h gives the contributions of produced hadrons with $\lambda_h = \pm 1$. Thus the ratio of the part multiplying λ_h and the $S_h = 0$ result gives the longitudinal polarization of the produced hadrons, which must lie between -1 and $+1$. Similarly, the ratio of the part multiplying S_{hT} and the $S_h = 0$ result gives the transverse polarization, again a number lying between -1 and $+1$. In many cases the final state hadron will not be a stable particle, e.g. a Λ . In that case the final state ($N\pi$ for the case of a Λ) is used to determine the spin vector S_h [37].

For the one-particle inclusive cross section we see one leading polarizing effect, namely for polarized leptons the longitudinal polarization of produced spin- $\frac{1}{2}$ particles is given by

$$\left(\text{longitudinal polarization} \right) = -\lambda_e \frac{C(y)}{2A(y)} \frac{\sum_{a,\bar{a}} e_a^2 G_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2 D_1^a(z_h)}. \quad (86)$$

At subleading order transverse polarization in the final state is induced given by

$$\left(\begin{array}{c} \text{transverse polarization} \\ \text{in lepton plane} \end{array} \right) = \lambda_e \frac{D(y)}{A(y)} \frac{2M_h}{z_h Q} \frac{\sum_{a,\bar{a}} e_a^2 G_T^a(z_h)}{\sum_{a,\bar{a}} e_a^2 D_1^a(z_h)}, \quad (87)$$

$$\left(\begin{array}{c} \text{transverse polarization} \\ \text{transverse to lepton plane} \end{array} \right) = \frac{C(y)D(y)}{A(y)} \frac{2M_h}{z_h Q} \frac{\sum_{a,\bar{a}} e_a^2 D_T^a(z_h)}{\sum_{a,\bar{a}} e_a^2 D_1^a(z_h)}, \quad (88)$$

where the lepton plane is spanned by l and P_2 . The in-plane polarization is proportional to the lepton polarization and is determined by the fragmentation function G_T . This function is the equivalent of the distribution function g_T . An out-of-plane polarization is found for unpolarized leptons, determined by the time-reversal-odd fragmentation function D_T . The asymmetry (88) was first discussed by Lu [38].

For the two-particle inclusive cross section in which one hadron has spin $\frac{1}{2}$, e.g. $e^+e^- \rightarrow \Lambda\pi X$, a longitudinal Λ polarization is induced,

$$\left(\text{longitudinal polarization} \right) = -\lambda_e \frac{C(y)}{2A(y)} \frac{\sum_{a,\bar{a}} e_a^2 G_1^{a \rightarrow \Lambda}(z_1) \bar{D}_1^{a \rightarrow \pi}(z_2)}{\sum_{a,\bar{a}} e_a^2 D_1^{a \rightarrow \Lambda}(z_1) \bar{D}_1^{a \rightarrow \pi}(z_2)}, \quad (89)$$

involving one polarized fragmentation function, namely $G_1^{a \rightarrow \Lambda}$. If both hadrons have spin $\frac{1}{2}$ a correlation between the polarizations of the two hadrons exists. The correlated longitudinal polarization involves $-\lambda_1 \lambda_2 A(y) \sum_{a,\bar{a}} e_a^2 G_1 \bar{G}_1$. The correlated transverse polarization involves $B(y) |S_{1T}| |S_{2T}| \cos(\phi_{S_1} + \phi_{S_2}) \sum_{a,\bar{a}} e_a^2 H_1 \bar{H}_1$ and provides a possibility to measure the transverse spin fragmentation function $H_1(z)$, the equivalent of the transverse spin distribution function h_1 [37]. For the two-particle inclusive cross section there are several single-spin asymmetries in unpolarized and polarized scattering, which among others give rise to twist-three fragmentation functions G_T , D_T , H_L , H , E and E_L , in principle each with characteristic final state polarization, but suppressed by M_h/Q .

6. Leading order asymmetries

Instead of integrating out the q_T dependence, we will now focus on the fully differential cross section, i.e. not integrated over transverse momentum ($\mathbf{P}_{1\perp} = -z_1 \mathbf{q}_T$). We will see that the transverse momentum dependent cross sections contain asymmetries, which would vanish upon integration. Some of those asymmetries appear at leading order, to which we restrict ourselves in this section. The subleading results can be obtained from the hadron tensor in Appendix B in a similar way.

First we consider the expression for both hadrons unpolarized,

$$\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 \mathbf{q}_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ A(y) \mathcal{F}[D_1 \bar{D}_1] + B(y) \cos(2\phi_1) \right. \\ \left. \times \mathcal{F} \left[\left(2\hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T \right) \frac{H_1^\perp \bar{H}_1^\perp}{M_1 M_2} \right] \right\}, \quad (90)$$

where we use the convolution notation

$$\mathcal{F}[D\bar{D}] \equiv \sum_{a,\bar{a}} e_a^2 \int d^2 \mathbf{k}_T d^2 \mathbf{p}_T \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) D^a(z_1, z_1^2 \mathbf{k}_T^2) \bar{D}^a(z_2, z_2^2 \mathbf{p}_T^2), \quad (91)$$

and $d\sigma^L = 0$ in this case. The angle ϕ_1 is the azimuthal angle of $\hat{\mathbf{h}}$, see Fig. 1. So we find that the number of produced hadrons has an azimuthal dependence,

$$N_{h_1 h_2}(z_1, z_2, \mathbf{q}_T, y) = z_1^2 z_2^2 \left\{ \mathcal{F}[D_1 \bar{D}_1] + \frac{B(y)}{A(y)} \cos(2\phi_1) \right. \\ \left. \times \mathcal{F} \left[\left(2\hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T \right) \frac{H_1^\perp \bar{H}_1^\perp}{M_1 M_2} \right] \right\} / \sum_{a,\bar{a}} e_a^2. \quad (92)$$

This asymmetry (the second term) has no analog in the Drell–Yan process or semi-inclusive lepton–hadron scattering, since it involves a product of two time-reversal-odd

functions. This new asymmetry goes with the same function H_1^\perp as appears in the Collins effect, multiplied with the similar time-reversal-odd function \bar{H}_1^\perp . We emphasize that this is a measurement in which no polarization of the produced hadrons is needed and the result is not suppressed by a factor of $1/Q$. This in contrast to the $\cos(2\phi)$ asymmetry found by Berger [21], which does not arise from time-reversal-odd functions. It is $1/Q^2$ suppressed and also arises in other processes.

Assuming for instance a Gaussian k_T dependence of the functions, the convolutions can be evaluated. The number of produced hadrons would then be

$$N_{h_1 h_2}(z_1, z_2, Q_T, y) = \mathcal{G}(Q_T; R) \sum_{a, \bar{a}} e_a^2 \left\{ D_1^a(z_1) \bar{D}_1^a(z_2) - \frac{B(y)}{A(y)} \cos(2\phi_1) \right. \\ \left. \times \frac{Q_T R^4}{M_1 M_2 R_1^2 R_2^2} H_1^{\perp a}(z_1) \bar{H}_1^{\perp a}(z_2) \right\} / \sum_{a, \bar{a}} e_a^2, \quad (93)$$

where

$$R^2 = R_1^2 R_2^2 / (R_1^2 + R_2^2),$$

$$D_1(z_1, k_T'^2) = D_1(z_1) R_1^2 \exp(-R_1^2 k_T'^2) / \pi z_1^2 \equiv D_1(z_1) \mathcal{G}(|k_T|; R_1) / z_1^2,$$

etc. For details see Ref. [12].

In case we consider the expression for hadron one polarized and hadron two unpolarized, we find the following additional terms:

$$\frac{d\sigma^O(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 q_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ \dots + B(y) \lambda_1 \sin(2\phi_1) \right. \\ \times \mathcal{F} \left[\left(2\hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T \right) \frac{H_{1L}^\perp \bar{H}_1^\perp}{M_1 M_2} \right] \\ - A(y) |S_{1T}| \sin(\phi_1 - \phi_{S_1}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \frac{D_{1T}^\perp \bar{D}_1}{M_1} \right] \\ + B(y) |S_{1T}| \sin(\phi_1 + \phi_{S_1}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{p}_T \frac{H_1 \bar{H}_1^\perp}{M_2} \right] \\ + B(y) |S_{1T}| \sin(3\phi_1 - \phi_{S_1}) \\ \left. \times \mathcal{F} \left[\left(4\hat{\mathbf{h}} \cdot \mathbf{p}_T (\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - 2\hat{\mathbf{h}} \cdot \mathbf{k}_T \mathbf{k}_T \cdot \mathbf{p}_T - \hat{\mathbf{h}} \cdot \mathbf{p}_T k_T^2 \right) \frac{H_{1T}^\perp \bar{H}_1^\perp}{2M_1^2 M_2} \right] \right\}, \quad (94)$$

$$\frac{d\sigma^L(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 q_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ -\lambda_e \frac{C(y)}{2} \lambda_1 \mathcal{F} [G_1 \bar{D}_1] \right. \\ \left. - \lambda_e \frac{C(y)}{2} |S_{1T}| \cos(\phi_1 - \phi_{S_1}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \frac{G_{1T} \bar{D}_1}{M_1} \right] \right\}. \quad (95)$$

Again there is a term which has no analog in semi-inclusive lepton-hadron scattering, namely the term with D_{1T}^\perp , which can be seen by comparison with the result obtained

in Ref. [39]. The term with $H_1 \bar{H}_1^\perp$ is the analog of the single-(transverse)-spin Collins effect. Note that it appears together with other single-transverse-spin asymmetries.

Conversely, one can consider hadron two polarized and hadron one unpolarized, which may be simpler from the experimental point of view, because one does not need to measure the transverse momentum and the transverse polarization of the same hadron. We find similar expressions in which all single-spin terms have, besides the obvious replacements, a sign change.

The leading order double spin asymmetries can be found in Appendix C and are useful in for instance the case of $e^+e^- \rightarrow \Lambda \bar{\Lambda} X$.

We like to point out that the transverse momentum dependence of some of the functions can be directly probed in the situation where hadron two is taken to be a jet, which in this back-to-back jet situation is equivalent to analyzing the azimuthal structure of hadrons inside a jet. Only \bar{D}_1 remains and is a delta function, so the convolutions can be evaluated exactly. In that case Eqs. (90), (94) and (95) taken together yield

$$\begin{aligned} \frac{d\sigma(e^+e^- \rightarrow h \text{ jet } X)}{d\Omega dz_h d^2q_T} = & \frac{3\alpha^2}{Q^2} z_h^2 \sum_{a,\bar{a}} e_a^2 \left\{ A(y) \left[D_1^a(z_h, z_h^2 Q_T^2) \right. \right. \\ & + |S_{hT}| \sin(\phi_h - \phi_{S_1}) \frac{Q_T}{M_h} D_{1T}^{\perp a}(z_h, z_h^2 Q_T^2) \left. \right] \\ & - \lambda_e \frac{C(y)}{2} \left[\lambda_h G_1^a(z_h, z_h^2 Q_T^2) \right. \\ & \left. \left. + |S_{hT}| \cos(\phi_h - \phi_{S_1}) \frac{Q_T}{M_h} G_{1T}^a(z_h, z_h^2 Q_T^2) \right] \right\}. \end{aligned} \quad (96)$$

This result means that there is a transverse polarization transverse to the hadron plane, which is proportional to the function D_{1T}^\perp , and a transverse polarization in the hadron plane, proportional to G_{1T} .

So one sees that by measuring q_T one can learn about the transverse momentum dependence of four functions in this particular case. There are no chiral-odd functions in this result, because they must be accompanied by a quark mass, which gives a result proportional to m/Q , so they are present in the subleading result.

7. Weighted cross sections

The expressions in the previous section contain convolutions, which are not the objects of interest, rather we want the (universal) fragmentation functions depending on z and k_T^2 . At the end of the previous section we discussed a situation in which the transverse momentum dependence of some of the functions could be extracted from the analysis of one jet. Below we will outline a way to obtain instead of the full transverse momentum dependence, the k_T^2 moments of the functions, defined as

$$F^{(n)}(z_1) = \int d^2 k'_T \left(\frac{k_T^2}{2M_1^2} \right)^n F(z_1, k_T'^2), \quad (97)$$

for a generic fragmentation function F . The lowest moment is the familiar k_T -integrated fragmentation function. By constructing appropriately weighted cross sections the convolutions result in products of such k_T^2 moments. The same k_T^2 moments for instance show up in semi-inclusive lepton-hadron scattering, in that case multiplied by the k_T^2 moments of the distribution functions [12]. In Section 5 we have presented the hadron tensor and cross section integrated over transverse photon momentum. A number of structures averaged out to zero, which are retained when the integration is weighted with an appropriate number of factors of q_T .

We find for the once-weighted cross sections, where we again show only the leading results, for the case that hadron two is unpolarized,

$$\begin{aligned} & \int d^2 q_T (q_T \cdot a) \frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 q_T} \\ &= \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 |a| \left\{ -A(y) |S_{1T}| \sin(\phi_{S_1} - \phi_a) \left(M_1 D_{1T}^{\perp(1)} \bar{D}_1 \right) \right. \\ & \quad - B(y) |S_{1T}| \sin(\phi_a + \phi_{S_1}) \left(M_2 H_1 \bar{H}_1^{\perp(1)} \right) \\ & \quad \left. - \lambda_e \frac{C(y)}{2} |S_{1T}| \cos(\phi_{S_1} - \phi_a) \left(M_1 G_{1T}^{(1)} \bar{D}_1 \right) \right\}. \end{aligned} \quad (98)$$

Constructing from this cross section the weighted one-particle inclusive cross section, by replacing \bar{D}_1 by a delta function, and considering the specific case $a = \hat{l}_\perp$, one finds

$$\begin{aligned} & \int d^2 q_T (q_T \cdot \hat{l}_\perp) \frac{d\sigma(e^+e^- \rightarrow hX)}{d\Omega dz_h d^2 q_T} \\ &= \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 \left\{ -A(y) |S_{hT}| \sin(\phi_{S_h}) M_h D_{1T}^{\perp(1)} \right. \\ & \quad \left. - \lambda_e \frac{C(y)}{2} |S_{hT}| \cos(\phi_{S_h}) M_h G_{1T}^{(1)} \right\}. \end{aligned} \quad (99)$$

These k_T^2 moments $D_{1T}^{\perp(1)}$ and $G_{1T}^{(1)}$ are related to the twist-three functions D_T and G_T via

$$D_T(z) = z^3 \frac{d}{dz} \left[\frac{D_{1T}^{\perp(1)}(z)}{z} \right], \quad (100)$$

$$G_T(z) = G_1(z) - z^3 \frac{d}{dz} \left[\frac{G_{1T}^{(1)}(z)}{z} \right], \quad (101)$$

respectively [12]. An experimental verification of these relations by comparing the above cross section to the one-particle inclusive results Eqs. (80) and (81) would be very interesting.

The twice-weighted cross section is

$$\begin{aligned}
 & \int d^2 q_T (q_T \cdot a) (q_T \cdot b) \frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 q_T} \\
 &= \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 |a||b| \left\{ -A(y) \cos(\phi_b - \phi_a) \left(M_2^2 D_1 \bar{D}_1^{(1)} + M_1^2 D_1^{(1)} \bar{D}_1 \right) \right. \\
 &\quad + 2B(y) M_1 M_2 \left[\cos(\phi_b + \phi_a) \left(H_1^{\perp(1)} \bar{H}_1^{\perp(1)} \right) \right. \\
 &\quad \left. \left. + \sin(\phi_b + \phi_a) \left(\lambda_1 H_{1L}^{\perp(1)} \bar{H}_1^{\perp(1)} \right) \right] \right. \\
 &\quad \left. + \lambda_e \frac{C(y)}{2} \cos(\phi_b - \phi_a) \left(+\lambda_1 M_2^2 G_1 \bar{D}_1^{(1)} + \lambda_1 M_1^2 G_1^{(1)} \bar{D}_1 \right) \right\}. \quad (102)
 \end{aligned}$$

In particular, one can use $(q_T \cdot \hat{l}_\perp)^2$, so one puts $a = b = \hat{l}_\perp$ in the above equation ($\phi_a = \phi_b = 0$), such that in the case that both hadrons are unpolarized

$$\begin{aligned}
 & \int d^2 q_T (q_T \cdot \hat{l}_\perp)^2 \frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 q_T} \\
 &= \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 \left\{ -A(y) \left(M_2^2 D_1 \bar{D}_1^{(1)} + M_1^2 D_1^{(1)} \bar{D}_1 \right) \right. \\
 &\quad \left. + 2B(y) M_1 M_2 H_1^{\perp(1)} \bar{H}_1^{\perp(1)} \right\}. \quad (103)
 \end{aligned}$$

Going back to the result for $N_{h_1 h_2}$ in Eq. (92), one sees that weighting that result only with $\cos(2\phi_1)$ would produce a convolution of H_1^\perp and \bar{H}_1^\perp , while the result above shows that including appropriate factors of $|q_T|$ produces a product of k_T^2 moments of fragmentation functions, in this case $H_1^{\perp(1)} \bar{H}_1^{\perp(1)}$. The k_T^2 moments can be used in other processes where they also occur. The above is an illustration of a general procedure.

In Appendices D and E we give the integrated once and twice-weighted hadron tensors, respectively, in case both hadrons are polarized. The twice-weighted result is only given to leading order.

8. Summary and conclusions

We have presented the complete tree-level result up to order $1/Q$ for inclusive two-hadron production in electron-positron annihilation. We consider the situation where the two hadrons belong to different, back-to-back jets. Polarization in the initial and final states is included for the case of spin- $\frac{1}{2}$ hadrons. In the case of spinless hadrons one will focus on the ones that are produced most abundantly, like the π 's and K 's, which

also serve to study the flavor dependence of fragmentation functions (see for instance Ref. [40]). For the case of spin- $\frac{1}{2}$ hadrons the A 's seem most appropriate due to their self-analyzing decays (see for instance Refs. [37,41]). Hadrons with higher spin, like the ρ 's, are not considered, because in that case a spin vector is not sufficient to describe the spin states.

We have restricted ourselves to the case of one photon exchange, since we are interested in power corrections which are, most likely, negligible in regions of Q^2 where the annihilation into Z bosons becomes important, i.e. LEP energies. In forthcoming work we will investigate the inclusion of Z 's in the leading part of our result taking into account the transverse momentum.

We have worked in a diagrammatic approach based upon to the one developed by Ellis, Furmanski and Petronzio for DIS. In this approach soft parts of a scattering process are treated as hadronic matrix elements of non-local operators. The soft parts occurring in the process under consideration are given by quark–quark and quark–gluon–quark correlation functions. The latter are necessary to achieve electromagnetic gauge invariance and are related to the quark–quark ones by use of the equations of motion. All soft non-perturbative physics is then parametrized by a set of fragmentation functions. The leading ones can be interpreted as quark decay functions.

We have done our calculations at tree level under the assumption that collinear divergences could be absorbed in the fragmentation functions and that factorization consequently holds. Moreover, for the interpretation of our results one should keep in mind that loop corrections, in general, will lead to finite order $(\alpha_s)^n$ corrections affecting the magnitude of observables and may also lead to non-zero contributions to observables not present at leading order (a well-known example is W_L in DIS).

Our results include among others the following:

- (i) We have considered the cross sections integrated over transverse momenta of the produced hadrons for both, polarized and unpolarized beams up to subleading order. Our result contains a few terms, which have been found previously in a leading order analysis for unpolarized e^+e^- annihilation [37].
- (ii) In particular, we have focused on the information obtainable by observing the transverse momentum of one of the produced hadrons (defined either relative to a jet axis or relative to the momentum of a hadron in the second jet). Although cross sections differential in transverse momentum are not easy measurable, they are of particular interest, since they contain leading order asymmetries, which would vanish upon integration.

We have found a number of new unpolarized, single and double spin asymmetries. Often they have no analogs in (semi-inclusive) DIS or the Drell–Yan process, since they involve products – or to be more specific, convolutions – of two time-reversal-odd fragmentation functions. In particular, the $\cos(2\phi)$ dependence discussed in Section 6 is most likely measurable, since it is not suppressed by powers of $1/Q$ and does not involve polarization, neither of the beams nor of the final states.

One-hadron inclusive measurements supplied with the additional determination

of the jet axis gives direct access to the transverse momentum dependence of some of the fragmentation functions.

- (iii) We have discussed how convolutions of fragmentation functions can be converted into products of their k_T^2 moments. This is achieved by appropriate weighting the integration over the transverse momentum dependence, in the spirit of a Fourier analysis. This is another way of retaining asymmetries, which would vanish upon (non-weighted) integration.

As a final note, it can be seen from our results which extra asymmetries will show up in, for instance, the Drell–Yan process if one allows for time-reversal-odd distribution functions. Such a single-spin asymmetry is discussed in Ref. [42]. However, the presence of time-reversal-odd distribution functions would require some factorization breaking mechanism, like the one discussed in Ref. [43].

In conclusion, we emphasize the possibilities provided by measuring azimuthal asymmetries and higher twist contributions in e^+e^- annihilation in order to learn more about the structure of hadrons.

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Appendix A. The fragmentation functions for a quark fragmenting into a quark

We consider the correlation function for the case of a quark (with momentum k) fragmenting into a quark (with momentum p and spin s), given by

$$\delta_{ij}(p, s; k) = u_i(k, s) \bar{u}_j(k, s) \delta^4(k - p) = \frac{1}{2} \left((\not{k} + m)(1 + \gamma_5 \not{s}) \right)_{ij} \delta^4(k - p), \quad (\text{A.1})$$

where the momentum and spin of the quark are parametrized as

$$k = \left[k^-, \frac{k_T^2 + m^2}{2k^-}, k_T \right], \quad (\text{A.2})$$

$$s = \left[\frac{\lambda_q k^-}{m}, -\frac{m \lambda_q}{2k^-} + \frac{k_T \cdot s_{qT}}{k^-} + \frac{\lambda_q k_T^2}{2mk^-}, s_{qT} + \frac{\lambda_q}{m} k_T \right] \quad (\text{A.3})$$

in terms of a quark light-cone helicity λ_q and a quark light-cone transverse polarization s_{qT} . The projections become for twist two

$$\delta^{[\gamma^-]}(k) = \frac{1}{2} \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.4})$$

$$\delta^{[\gamma^- \gamma_5]}(k) = \frac{1}{2} \lambda_q \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.5})$$

$$\delta^{[i\sigma^- \gamma_5]}(k) = \frac{1}{2} s_{qT}^i \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.6})$$

where $z = p^-/k^-$. For twist three we get

$$\delta^{[11]}(k) = \frac{m}{2k^-} \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.7})$$

$$\delta^{[\gamma^i]}(k) = \frac{\mathbf{k}_T^i}{2k^-} \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.8})$$

$$\delta^{[\gamma^i \gamma_5]}(k) = \frac{(m s_{qT}^i + \lambda_q \mathbf{k}_T^i)}{2k^-} \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.9})$$

$$\delta^{[i\sigma^{jj} \gamma_5]}(k) = \frac{s_{qT}^i \mathbf{k}_T^j - \mathbf{k}_T^i s_{qT}^j}{2k^-} \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T), \quad (\text{A.10})$$

$$\delta^{[i\sigma^{-+} \gamma_5]}(k) = \frac{m \lambda_q - \mathbf{k}_T \cdot \mathbf{s}_{qT}}{2k^-} \delta(z-1) \delta^2(\mathbf{k}_T - \mathbf{p}_T). \quad (\text{A.11})$$

Appendix B. The complete expression for the hadron tensor

The full expressions for the symmetric and antisymmetric parts of the hadron tensor are (expressed in the perpendicular frame defined in Section 2)

$$\begin{aligned} \mathcal{W}_S^{\mu\nu} = & 12e^2 z_1 z_2 \int d^2 \mathbf{k}_T d^2 \mathbf{p}_T \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) \\ & \times \left\{ -g_{\perp}^{\mu\nu} \left[D_1 \bar{D}_1 - G_{1s} \bar{G}_{1s} + \frac{\epsilon_{\perp}^{\rho\sigma} k_{\perp\rho} S_{1\perp\sigma}}{M_1} D_{1T}^{\perp} \bar{D}_1 - \frac{\epsilon_{\perp}^{\rho\sigma} p_{\perp\rho} S_{2\perp\sigma}}{M_2} D_1 \bar{D}_{1T}^{\perp} \right. \right. \\ & \left. \left. - \frac{\mathbf{k}_{\perp} \cdot \mathbf{p}_{\perp} S_{1\perp} \cdot S_{2\perp} - \mathbf{p}_{\perp} \cdot S_{1\perp} \mathbf{k}_{\perp} \cdot S_{2\perp}}{M_1 M_2} D_{1T}^{\perp} \bar{D}_{1T}^{\perp} \right] \right. \\ & - \left(S_{1\perp}^{\{\mu} S_{2\perp}^{\nu\}} + g_{\perp}^{\mu\nu} S_{1\perp} \cdot S_{2\perp} \right) H_{1T} \bar{H}_{1T} \\ & - \frac{k_{\perp}^{\{\mu} p_{\perp}^{\nu\}} + g_{\perp}^{\mu\nu} \mathbf{k}_{\perp} \cdot \mathbf{p}_{\perp}}{M_1 M_2} \left(H_{1s}^{\perp} \bar{H}_{1s}^{\perp} + H_1^{\perp} \bar{H}_1^{\perp} \right) \\ & - \frac{k_{\perp}^{\{\mu} S_{2\perp}^{\nu\}} + g_{\perp}^{\mu\nu} \mathbf{k}_{\perp} \cdot S_{2\perp}}{M_1} H_{1s}^{\perp} \bar{H}_{1T} - \frac{p_{\perp}^{\{\mu} S_{1\perp}^{\nu\}} + g_{\perp}^{\mu\nu} \mathbf{p}_{\perp} \cdot S_{1\perp}}{M_2} H_{1T} \bar{H}_{1s}^{\perp} \\ & + \frac{k_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} p_{\perp\rho} + p_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} k_{\perp\rho}}{2M_1 M_2} \left(H_{1s}^{\perp} \bar{H}_1^{\perp} - H_1^{\perp} \bar{H}_{1s}^{\perp} \right) \\ & \left. - \frac{k_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} S_{2\perp\rho} + S_{2\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} k_{\perp\rho}}{2M_1} H_1^{\perp} \bar{H}_{1T} \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{p_{\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} S_{1\perp\rho} + S_{1\perp}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} p_{\perp\rho}}{2M_2} H_{1T} \bar{H}_1^{\perp} \\
& + 2 \frac{\hat{z}^{\{\mu} k_{\perp}^{\nu\}}}{Q} \left[\frac{\bar{D}_{\perp}^{\perp}}{z_1} \bar{D}_1 - \frac{S_{1\perp} \cdot S_{2\perp}}{M_1} M_2 D_{1T}^{\perp} \frac{\bar{D}_T}{z_2} - \lambda_2 \frac{p_{\perp} \cdot S_{1\perp}}{M_1} D_{1T}^{\perp} \frac{\bar{D}_L^{\perp}}{z_2} \right. \\
& + \frac{k_{\perp} \cdot p_{\perp} S_{1\perp} \cdot S_{2\perp} - p_{\perp} \cdot S_{1\perp} k_{\perp} \cdot S_{2\perp}}{M_1 M_2} D_{1T}^{\perp} \bar{D}_{1T}^{\perp} - \frac{\bar{G}_s^{\perp}}{z_1} \bar{G}_{1s} \\
& + \frac{p_{\perp} \cdot S_{1\perp}}{M_2} \frac{\bar{H}_T^{\perp}}{z_1} \bar{H}_{1s}^{\perp} + S_{1\perp} \cdot S_{2\perp} \frac{\bar{H}_T^{\perp}}{z_1} \bar{H}_{1T} - \frac{M_2}{M_1} \left(H_{1s}^{\perp} \frac{\bar{H}_s}{z_2} + H_1^{\perp} \frac{\bar{H}}{z_2} \right) \left. \right] \\
& + 2 \frac{\hat{z}^{\{\mu} p_{\perp}^{\nu\}}}{Q} \left[-D_1 \frac{\bar{D}_{\perp}^{\perp}}{z_2} - \frac{k_{\perp}^2 S_{1\perp} \cdot S_{2\perp}}{2M_1 M_2} D_{1T}^{\perp} \bar{D}_{1T}^{\perp} + \lambda_1 \frac{k_{\perp} \cdot S_{2\perp}}{M_2} \frac{\bar{D}_L^{\perp}}{z_1} \bar{D}_{1T}^{\perp} \right. \\
& + \frac{M_1}{M_2} S_{1\perp} \cdot S_{2\perp} \frac{\bar{D}_T}{z_1} \bar{D}_{1T}^{\perp} + G_{1s} \frac{\bar{G}_s^{\perp}}{z_2} - \frac{k_{\perp} \cdot S_{2\perp}}{M_1} H_{1s}^{\perp} \frac{\bar{H}_T}{z_2} - S_{1\perp} \cdot S_{2\perp} H_{1T} \frac{\bar{H}_T}{z_2} \\
& + \frac{M_1}{M_2} \left(\frac{\bar{H}}{z_1} \bar{H}_1^{\perp} + \frac{\bar{H}_s}{z_1} \bar{H}_{1s}^{\perp} \right) \left. \right] \\
& + 2 \frac{\hat{z}^{\{\mu} S_{1\perp}^{\nu\}}}{Q} \left[\lambda_2 \frac{k_{\perp} \cdot p_{\perp}}{M_1} D_{1T}^{\perp} \frac{\bar{D}_L^{\perp}}{z_2} + \frac{M_2}{M_1} k_{\perp} \cdot S_{2\perp} D_{1T}^{\perp} \frac{\bar{D}_T}{z_2} - M_1 \frac{\bar{G}_T'}{z_1} \bar{G}_{1s} \right. \\
& - \frac{k_{\perp} \cdot p_{\perp}}{M_2} \frac{\bar{H}_T^{\perp}}{z_1} \bar{H}_{1s}^{\perp} - M_2 H_{1T} \frac{\bar{H}_s}{z_2} - k_{\perp} \cdot S_{2\perp} \frac{\bar{H}_T^{\perp}}{z_1} \bar{H}_{1T} \left. \right] \\
& + 2 \frac{\hat{z}^{\{\mu} S_{2\perp}^{\nu\}}}{Q} \left[-\lambda_1 \frac{k_{\perp} \cdot p_{\perp}}{M_2} \frac{\bar{D}_L^{\perp}}{z_1} \bar{D}_{1T}^{\perp} + \frac{p_{\perp} \cdot S_{1\perp} k_{\perp}^2}{2M_1 M_2} D_{1T}^{\perp} \bar{D}_{1T}^{\perp} \right. \\
& - \frac{M_1}{M_2} p_{\perp} \cdot S_{1\perp} \frac{\bar{D}_T}{z_1} \bar{D}_{1T}^{\perp} + M_2 G_{1s} \frac{\bar{G}_T'}{z_2} + \frac{k_{\perp} \cdot p_{\perp}}{M_1} H_{1s}^{\perp} \frac{\bar{H}_T}{z_2} + p_{\perp} \cdot S_{1\perp} H_{1T} \frac{\bar{H}_T}{z_2} \\
& + M_1 \frac{\bar{H}_s}{z_1} \bar{H}_{1T} \left. \right] \\
& + 2 \frac{\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} k_{\perp\rho}}{Q} \left[\lambda_1 \frac{\bar{D}_L^{\perp}}{z_1} \bar{D}_1 - \frac{p_{\perp} \cdot S_{1\perp}}{M_1} D_{1T}^{\perp} \frac{\bar{D}_{\perp}^{\perp}}{z_2} - \frac{k_{\perp} \cdot S_{1\perp}}{M_1} D_{1T}^{\perp} \bar{D}_1 \right. \\
& + \frac{m}{M_1} H_1^{\perp} \bar{G}_{1s} - \frac{p_{\perp} \cdot S_{1\perp}}{M_2} \frac{\bar{H}_T^{\perp}}{z_1} \bar{H}_1^{\perp} + \frac{M_2}{M_1} \left(H_{1s}^{\perp} \frac{\bar{H}}{z_2} - H_1^{\perp} \frac{\bar{H}_s}{z_2} \right) \left. \right] \\
& + 2 \frac{\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} p_{\perp\rho}}{Q} \left[\lambda_2 D_1 \frac{\bar{D}_L^{\perp}}{z_2} - \frac{k_{\perp} \cdot S_{2\perp}}{M_2} \frac{\bar{D}_{\perp}^{\perp}}{z_1} \bar{D}_{1T}^{\perp} - \frac{k_{\perp} \cdot S_{2\perp}}{M_1} H_1^{\perp} \frac{\bar{H}_T}{z_2} \right. \\
& + \frac{M_1}{M_2} \left(\frac{\bar{H}}{z_1} \bar{H}_{1s}^{\perp} - \frac{\bar{H}_s}{z_1} \bar{H}_1^{\perp} \right) \left. \right] \\
& + 2 \frac{\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\}\rho} S_{1\perp\rho}}{Q} \left[M_1 \frac{\bar{D}_T}{z_1} \bar{D}_1 + \frac{k_{\perp} \cdot p_{\perp}}{M_2} \frac{\bar{H}_T^{\perp}}{z_1} \bar{H}_1^{\perp} + \frac{k_{\perp} \cdot p_{\perp}}{M_1} D_{1T}^{\perp} \frac{\bar{D}_{\perp}^{\perp}}{z_2} \right. \\
& + \frac{k_{\perp}^2}{2M_1} D_{1T}^{\perp} \bar{D}_1 + M_2 H_{1T} \frac{\bar{H}}{z_2} \left. \right]
\end{aligned}$$

$$\begin{aligned}
& + 2 \frac{\hat{z}^{[\mu} \epsilon_{\perp}^{\nu]} \rho S_{2\perp\rho}}{Q} \left[M_2 D_1 \frac{\bar{D}_T}{z_2} + M_1 \frac{\bar{H}}{z_1} \bar{H}_{1T} + \frac{\mathbf{k}_{\perp} \cdot \mathbf{p}_{\perp}}{M_2} \frac{\bar{D}^{\perp}}{z_1} \bar{D}_{1T}^{\perp} \right. \\
& \left. + \frac{\mathbf{k}_{\perp} \cdot \mathbf{p}_{\perp}}{M_1} H_1^{\perp} \frac{\bar{H}_T^{\perp}}{z_2} \right] \Big\} \quad (\text{B.1})
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{W}_A^{\mu\nu} = & 12e^2 z_1 z_2 \int d^2 \mathbf{k}_T d^2 \mathbf{p}_T \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) \\
& \times \left\{ + i \epsilon_{\perp}^{\mu\nu} \left[G_{1s} \bar{D}_1 - D_1 \bar{G}_{1s} \right] - i p_{\perp}^{[\mu} S_{2\perp}^{\nu]} \frac{1}{M_2} G_{1s} \bar{D}_{1T}^{\perp} - i k_{\perp}^{[\mu} S_{1\perp}^{\nu]} \frac{1}{M_1} D_{1T}^{\perp} \bar{G}_{1s} \right. \\
& + 2i \frac{\hat{z}^{[\mu} k_{\perp}^{\nu]}}{Q} \left[\lambda_1 \frac{\bar{D}_L^{\perp}}{z_1} \bar{G}_{1s} - \frac{\mathbf{k}_{\perp} \cdot \mathbf{S}_{1\perp}}{M_1} D_{1T}^{\perp} \bar{G}_{1s} \right. \\
& - \frac{\mathbf{S}_{1\perp} \cdot \mathbf{S}_{2\perp}}{M_1} \left(M_2 D_{1T}^{\perp} \frac{\bar{G}_T'}{z_2} - m D_{1T}^{\perp} \bar{H}_{1T} \right) \\
& - \frac{\mathbf{p}_{\perp} \cdot \mathbf{S}_{1\perp}}{M_1} \left(D_{1T}^{\perp} \frac{\bar{G}_s^{\perp}}{z_2} - \frac{m}{M_2} D_{1T}^{\perp} \bar{H}_{1s}^{\perp} \right) + \frac{m}{M_1} H_1^{\perp} \bar{D}_1 \\
& - \frac{M_2}{M_1} \left(H_{1s}^{\perp} \frac{\bar{E}_s}{z_2} + H_1^{\perp} \frac{\bar{E}}{z_2} \right) \Big] \\
& + 2i \frac{\hat{z}^{[\mu} p_{\perp}^{\nu]}}{Q} \left[-\lambda_2 G_{1s} \frac{\bar{D}_L^{\perp}}{z_2} + \frac{\mathbf{k}_{\perp} \cdot \mathbf{S}_{2\perp}}{M_2} \frac{\bar{G}_s^{\perp}}{z_1} \bar{D}_{1T}^{\perp} + \frac{\mathbf{S}_{1\perp} \cdot \mathbf{S}_{2\perp}}{M_2} M_1 \frac{\bar{G}_T'}{z_1} \bar{D}_{1T}^{\perp} \right. \\
& + \frac{M_1}{M_2} \left(\frac{\bar{E}_s}{z_1} \bar{H}_{1s}^{\perp} + \frac{\bar{E}}{z_1} \bar{H}_1^{\perp} \right) \Big] \\
& + 2i \frac{\hat{z}^{[\mu} S_{1\perp}^{\nu]}}{Q} \left[M_1 \frac{\bar{D}_T}{z_1} \bar{G}_{1s} + \frac{\mathbf{k}_{\perp} \cdot \mathbf{S}_{2\perp}}{M_1} \left(M_2 D_{1T}^{\perp} \frac{\bar{G}_T'}{z_2} - m D_{1T}^{\perp} \bar{H}_{1T} \right) \right. \\
& + \frac{\mathbf{k}_{\perp}^2}{2M_1} D_{1T}^{\perp} \bar{G}_{1s} + \frac{\mathbf{k}_{\perp} \cdot \mathbf{p}_{\perp}}{M_1 M_2} \left(M_2 D_{1T}^{\perp} \frac{\bar{G}_s^{\perp}}{z_2} - m D_{1T}^{\perp} \bar{H}_{1s}^{\perp} \right) - M_2 H_{1T} \frac{\bar{E}_s}{z_2} \Big] \\
& + 2i \frac{\hat{z}^{[\mu} S_{2\perp}^{\nu]}}{Q} \left[-M_2 G_{1s} \frac{\bar{D}_T}{z_2} - \frac{\mathbf{p}_{\perp} \cdot \mathbf{S}_{1\perp}}{M_2} M_1 \frac{\bar{G}_T'}{z_1} \bar{D}_{1T}^{\perp} - \frac{\mathbf{k}_{\perp} \cdot \mathbf{p}_{\perp}}{M_2} \frac{\bar{G}_s^{\perp}}{z_1} \bar{D}_{1T}^{\perp} \right. \\
& + M_1 \frac{\bar{E}_s}{z_1} \bar{H}_{1T} \Big] \\
& + 2i \frac{\hat{z}^{[\mu} \epsilon_{\perp}^{\nu]} \rho k_{\perp\rho}}{Q} \left[\frac{\bar{G}_s^{\perp}}{z_1} \bar{D}_1 - \frac{\bar{D}^{\perp}}{z_1} \bar{G}_{1s} - \frac{\mathbf{p}_{\perp} \cdot \mathbf{S}_{1\perp}}{M_1 M_2} m D_{1T}^{\perp} \bar{H}_1^{\perp} \right. \\
& + \frac{M_2}{M_1} \left(H_{1s}^{\perp} \frac{\bar{E}}{z_2} - H_1^{\perp} \frac{\bar{E}_s}{z_2} \right) \Big] \\
& + 2i \frac{\hat{z}^{[\mu} \epsilon_{\perp}^{\nu]} \rho p_{\perp\rho}}{Q} \left[D_1 \frac{\bar{G}_s^{\perp}}{z_2} - G_{1s} \frac{\bar{D}^{\perp}}{z_2} - \frac{\mathbf{k}_{\perp} \cdot \mathbf{S}_{2\perp}}{M_1 M_2} m H_1^{\perp} \bar{D}_{1T}^{\perp} \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{M_1}{M_2} \left(\frac{\tilde{E}}{z_1} \overline{H}_{1s}^\perp - \frac{\tilde{E}_s}{z_1} \overline{H}_1^\perp \right) \Bigg] \\
& + 2i \frac{\hat{z}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{1\perp\rho}}{Q} \left[M_1 \frac{\tilde{G}'_T}{z_1} \overline{D}_1 + \frac{\mathbf{k}_\perp \cdot \mathbf{p}_\perp}{M_1 M_2} m D_{1T}^\perp \overline{H}_1^\perp + M_2 H_{1T} \frac{\tilde{E}}{z_2} \right] \\
& + 2i \frac{\hat{z}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{2\perp\rho}}{Q} \left[M_2 D_1 \frac{\tilde{G}'_T}{z_2} + \frac{\mathbf{k}_\perp \cdot \mathbf{p}_\perp}{M_1 M_2} m H_1^\perp \overline{D}_{1T}^\perp + M_1 \frac{\tilde{E}}{z_1} \overline{H}_{1T} \right] \Bigg\}. \quad (\text{B.2})
\end{aligned}$$

Appendix C. Double spin asymmetries

In this appendix we give the azimuthal dependences of double spin asymmetries, as can be observed, for instance, in $\Lambda\bar{\Lambda}$ production by determination of the polarizations of both observed hadrons. The spin-independent and single-spin dependent parts of the cross section are given in Eqs. (90), (94) and (95).

$$\begin{aligned}
\frac{d\sigma^{(2)}(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\mathbf{q}_T} &= \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ -\frac{A(y)}{2} \lambda_1 \lambda_2 \mathcal{F} [G_1 \overline{G}_1] \right. \\
&- A(y) \lambda_1 |S_{2T}| \cos(\phi_1 - \phi_{S_2}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{p}_T \frac{G_1 \overline{G}_{1T}}{M_2} \right] \\
&+ \frac{A(y)}{2} |S_{1T}| |S_{2T}| \cos(2\phi_1 - \phi_{S_1} - \phi_{S_2}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T \frac{D_{1T}^\perp \overline{D}_{1T}^\perp - G_{1T} \overline{G}_{1T}}{M_1 M_2} \right] \\
&- \frac{A(y)}{2} |S_{1T}| |S_{2T}| \cos(\phi_1 - \phi_{S_1}) \cos(\phi_1 - \phi_{S_2}) \mathcal{F} \left[\mathbf{k}_T \cdot \mathbf{p}_T \frac{D_{1T}^\perp \overline{D}_{1T}^\perp}{M_1 M_2} \right] \\
&- \frac{A(y)}{2} |S_{1T}| |S_{2T}| \sin(\phi_1 - \phi_{S_1}) \sin(\phi_1 - \phi_{S_2}) \mathcal{F} \left[\mathbf{k}_T \cdot \mathbf{p}_T \frac{G_{1T} \overline{G}_{1T}}{M_1 M_2} \right] \\
&+ \frac{B(y)}{2} |S_{1T}| |S_{2T}| \cos(\phi_{S_1} + \phi_{S_2}) \mathcal{F} [H_1 \overline{H}_1] \\
&+ B(y) \lambda_1 |S_{2T}| \cos(\phi_1 + \phi_{S_2}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \frac{H_{1L}^\perp \overline{H}_1}{M_1} \right] \\
&+ \frac{B(y)}{2} \lambda_1 \lambda_2 \cos(2\phi_1) \mathcal{F} \left[\left(2\hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T \right) \frac{H_{1L}^\perp \overline{H}_{1L}^\perp}{M_1 M_2} \right] \\
&+ \frac{B(y)}{2} |S_{1T}| |S_{2T}| \cos(2\phi_1 - \phi_{S_1} + \phi_{S_2}) \mathcal{F} \left[\left(2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - \mathbf{k}_T^2 \right) \frac{H_{1T}^\perp \overline{H}_1}{M_1} \right] \\
&+ \frac{B(y)}{2} \lambda_2 |S_{1T}| \cos(3\phi_1 - \phi_{S_1}) \\
&\times \mathcal{F} \left[\left(4\hat{\mathbf{h}} \cdot \mathbf{p}_T (\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - 2\hat{\mathbf{h}} \cdot \mathbf{k}_T \mathbf{k}_T \cdot \mathbf{p}_T - \hat{\mathbf{h}} \cdot \mathbf{p}_T \mathbf{k}_T^2 \right) \frac{H_{1T}^\perp \overline{H}_{1L}^\perp}{M_1^2 M_2} \right] \\
&+ \frac{B(y)}{8} |S_{1T}| |S_{2T}| \cos(4\phi_1 - \phi_{S_1} - \phi_{S_2})
\end{aligned}$$

$$\begin{aligned}
& \times \mathcal{F} \left[\left(8(\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 (\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 - 4\mathbf{k}_T \cdot \mathbf{p}_T \hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - 2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 \mathbf{p}_T^2 \right. \right. \\
& \left. \left. - 2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 \mathbf{k}_T^2 + (\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 (\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 \right) \frac{H_{1T}^\perp \bar{H}_{1T}^\perp}{M_1^2 M_2^2} \right] \\
& - \lambda_e \frac{C(y)}{2} \lambda_1 |S_{2T}| \sin(\phi_1 - \phi_{S_2}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{p}_T \frac{G_{1T} \bar{D}_{1T}^\perp}{M_2} \right] \\
& - \lambda_e \frac{C(y)}{4} |S_{1T}| |S_{2T}| \sin(2\phi_1 - \phi_{S_1} - \phi_{S_2}) \mathcal{F} \left[\hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T \frac{D_{1T}^\perp \bar{G}_{1T} + G_{1T} \bar{D}_{1T}^\perp}{M_1 M_2} \right] \\
& - \lambda_e \frac{C(y)}{2} |S_{1T}| |S_{2T}| \sin(\phi_1 - \phi_{S_2}) \cos(\phi_1 - \phi_{S_1}) \mathcal{F} \left[\mathbf{k}_T \cdot \mathbf{p}_T \frac{D_{1T}^\perp \bar{G}_{1T}}{M_1 M_2} \right] \\
& + \left(\begin{matrix} 1 \longleftrightarrow 2 \\ \mathbf{p} \longleftrightarrow \mathbf{k} \end{matrix} \right) \Bigg\}. \tag{C.1}
\end{aligned}$$

Appendix D. Integrated once-weighted hadron tensor

We display the hadron tensor weighted with the factor $(\mathbf{q}_T \cdot \mathbf{a})$ and integrated over the transverse photon momentum. The vector \mathbf{a} is an arbitrary vector like, for instance, $\hat{\mathbf{l}}_\perp$.

$$\begin{aligned}
\int d^2 \mathbf{q}_T (\mathbf{q}_T \cdot \mathbf{a}) \mathcal{W}_S^{\mu\nu} &= 12e^2 z_1 z_2 \left\{ -g_\perp^{\mu\nu} \left[-\lambda_1 \mathbf{a} \cdot \mathbf{S}_{2\perp} M_2 G_{1T} \bar{G}_{1T}^{(1)} \right. \right. \\
& \left. \left. - \lambda_2 \mathbf{a} \cdot \mathbf{S}_{1\perp} M_1 G_{1T}^{(1)} \bar{G}_1 + \epsilon_\perp^{\rho\sigma} a_\rho S_{1\perp\sigma} M_1 D_{1T}^{\perp(1)} \bar{D}_1 - \epsilon_\perp^{\rho\sigma} a_\rho S_{2\perp\sigma} M_2 D_1 \bar{D}_{1T}^{\perp(1)} \right] \right. \\
& - \left(S_{1\perp}^{\{\mu} a^{\nu\}} + g_\perp^{\mu\nu} \mathbf{a} \cdot \mathbf{S}_{1\perp} \right) \lambda_2 M_2 H_1 \bar{H}_{1L}^{\perp(1)} \\
& - \left(S_{2\perp}^{\{\mu} a^{\nu\}} + g_\perp^{\mu\nu} \mathbf{a} \cdot \mathbf{S}_{2\perp} \right) \lambda_1 M_1 H_{1L}^{\perp(1)} \bar{H}_1 \\
& + \left(a^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{1\perp\rho} + S_{1\perp}^{\{\mu} \epsilon_\perp^{\nu\}\rho} a_\rho \right) \frac{M_2}{2} H_1 \bar{H}_1^{\perp(1)} \\
& - \left(a^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{2\perp\rho} + S_{2\perp}^{\{\mu} \epsilon_\perp^{\nu\}\rho} a_\rho \right) \frac{M_1}{2} H_1^{\perp(1)} \bar{H}_1 \\
& + 2 \frac{\hat{z}^{\{\mu} S_{1\perp}^{\nu\}}}{Q} \mathbf{a} \cdot \mathbf{S}_{2\perp} \left[M_1 M_2 D_{1T}^{\perp(1)} \frac{\bar{D}_T}{z_2} - M_1 M_2 \frac{\tilde{G}_T}{z_1} \bar{G}_{1T}^{(1)} - M_2^2 H_1 \frac{\bar{H}_T^{(1)}}{z_2} \right. \\
& \left. - M_1^2 \frac{\tilde{H}_T^{\perp(1)}}{z_1} \bar{H}_1 \right] \\
& + 2 \frac{\hat{z}^{\{\mu} S_{2\perp}^{\nu\}}}{Q} \mathbf{a} \cdot \mathbf{S}_{1\perp} \left[-M_1 M_2 \frac{\tilde{D}_T}{z_1} \bar{D}_{1T}^{\perp(1)} + M_1 M_2 G_{1T}^{(1)} \frac{\bar{G}_T}{z_2} + M_2^2 H_1 \frac{\bar{H}_T^{\perp(1)}}{z_2} \right. \\
& \left. + M_1^2 \frac{\tilde{H}_T^{(1)}}{z_1} \bar{H}_1 \right] \Bigg\}
\end{aligned}$$

$$\begin{aligned}
& -2 \frac{\hat{z}^{\{\mu} a^{\nu\}}}{Q} \left[M_2^2 D_1 \frac{\bar{D}^{\perp(1)}}{z_2} - M_1^2 \frac{\tilde{D}^{\perp(1)}}{z_1} \bar{D}_1 \right. \\
& - M_1 M_2 \mathbf{S}_{1\perp} \cdot \mathbf{S}_{2\perp} \left(\frac{\tilde{D}_T}{z_1} \bar{D}_{1T}^{\perp(1)} - D_{1T}^{\perp(1)} \frac{\bar{D}_T}{z_2} \right) \\
& + \lambda_1 \lambda_2 \left(M_1^2 \frac{\tilde{G}_L^{\perp(1)}}{z_1} \bar{G}_1 - M_2^2 G_1 \frac{\bar{G}_L^{\perp(1)}}{z_2} \right) \\
& + \lambda_1 \lambda_2 M_1 M_2 \left(H_{1L}^{\perp(1)} \frac{\bar{H}_L}{z_2} - \frac{\tilde{H}_L}{z_1} \bar{H}_{1L}^{\perp(1)} \right) + M_1 M_2 \left(H_1^{\perp(1)} \frac{\bar{H}}{z_2} - \frac{\tilde{H}}{z_1} \bar{H}_1^{\perp(1)} \right) \\
& - \mathbf{S}_{1\perp} \cdot \mathbf{S}_{2\perp} \left(M_1^2 \frac{\tilde{H}_T^{\perp(1)}}{z_1} \bar{H}_1 - M_2^2 H_1 \frac{\bar{H}_T^{\perp(1)}}{z_2} \right) \left. \right] \\
& - 2 \frac{\hat{z}^{\{\mu} \epsilon_{\perp}^{\nu\} \rho} a_{\rho}}{Q} \left[-\lambda_1 M_1^2 \frac{\tilde{D}_L^{\perp(1)}}{z_1} \bar{D}_1 - \lambda_2 M_2^2 D_1 \frac{\bar{D}_L^{\perp(1)}}{z_2} - \lambda_2 m M_1 H_1^{\perp(1)} \bar{G}_1 \right. \\
& + \lambda_1 M_1 M_2 \left(\frac{\tilde{H}_L}{z_1} \bar{H}_1^{\perp(1)} - H_{1L}^{\perp(1)} \frac{\bar{H}}{z_2} \right) + \lambda_2 M_1 M_2 \left(H_1^{\perp(1)} \frac{\bar{H}_L}{z_2} - \frac{\tilde{H}}{z_1} \bar{H}_{1L}^{\perp(1)} \right) \left. \right] \Bigg\} \\
& \hspace{15em} \text{(D.1)}
\end{aligned}$$

and

$$\begin{aligned}
& \int d^2 q_T (q_T \cdot a) \mathcal{W}_A^{\mu\nu} = 12 e^2 z_1 z_2 \left\{ i \epsilon_{\perp}^{\mu\nu} \left[M_1 a \cdot \mathbf{S}_{1\perp} G_{1T}^{(1)} \bar{D}_1 - M_2 a \cdot \mathbf{S}_{2\perp} D_1 \bar{G}_{1T}^{(1)} \right] \right. \\
& + i S_{1\perp}^{\perp(\mu} a^{\nu)} \left[\lambda_2 M_1 D_{1T}^{\perp(1)} \bar{G}_1 \right] + i S_{2\perp}^{\perp(\mu} a^{\nu)} \left[\lambda_1 M_2 G_1 \bar{D}_{1T}^{\perp(1)} \right] \\
& + i 2 \frac{\hat{z}^{\perp(\mu} S_{1\perp}^{\nu)}}{Q} a \cdot \mathbf{S}_{2\perp} \left[M_1 M_2 D_{1T}^{\perp(1)} \frac{\bar{G}_T}{z_2} + M_1 M_2 \frac{\tilde{D}_T}{z_1} \bar{G}_{1T}^{(1)} - m M_1 D_{1T}^{\perp(1)} \bar{H}_1 \right. \\
& \left. - M_2^2 H_1 \frac{\bar{E}_T^{(1)}}{z_2} \right] \\
& + i 2 \frac{\hat{z}^{\perp(\mu} S_{2\perp}^{\nu)}}{Q} a \cdot \mathbf{S}_{1\perp} \left[-M_1 M_2 G_{1T}^{(1)} \frac{\bar{D}_T}{z_2} - M_1 M_2 \frac{\tilde{G}_T}{z_1} \bar{D}_{1T}^{\perp(1)} + M_1^2 \frac{\tilde{E}_T}{z_1} (1) \bar{H}_1 \right] \\
& - i 2 \frac{\hat{z}^{\perp(\mu} a^{\nu)}}{Q} \left[\lambda_1 \lambda_2 \left(M_2^2 G_1 \frac{\bar{D}_L^{\perp(1)}}{z_2} - M_1^2 \frac{\tilde{D}_L^{\perp(1)}}{z_1} \bar{G}_1 \right) \right. \\
& - M_1 M_2 \mathbf{S}_{1\perp} \cdot \mathbf{S}_{2\perp} \left(\frac{\tilde{G}_T}{z_1} \bar{D}_{1T}^{\perp(1)} - D_{1T}^{\perp(1)} \frac{\bar{G}_T}{z_2} \right) \\
& - m M_1 \left(\mathbf{S}_{1\perp} \cdot \mathbf{S}_{2\perp} D_{1T}^{\perp(1)} \bar{H}_1 + H_1^{\perp(1)} \bar{D}_1 \right) + M_1 M_2 \left(H_1^{\perp(1)} \frac{\bar{E}}{z_2} - \frac{\tilde{E}}{z_1} \bar{H}_1^{\perp(1)} \right) \\
& \left. + \lambda_1 \lambda_2 M_1 M_2 \left(H_{1L}^{\perp(1)} \frac{\bar{E}_L}{z_2} - \frac{\tilde{E}_L}{z_1} \bar{H}_{1L}^{\perp(1)} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& -i2 \frac{\hat{z}^{[\mu} \epsilon_{\perp}^{\nu] \rho} a_{\rho}}{Q} \left[\lambda_1 M_2^2 G_1 \frac{\bar{D}^{\perp(1)}}{z_2} + \lambda_2 M_1^2 \frac{\tilde{D}^{\perp(1)}}{z_1} \bar{G}_1 - \lambda_1 M_1^2 \frac{\tilde{G}_L^{\perp(1)}}{z_1} \bar{D}_1 \right. \\
& - \lambda_2 M_2^2 D_1 \frac{\bar{G}_L^{\perp(1)}}{z_2} \\
& \left. + M_1 M_2 \left(\lambda_1 \frac{\tilde{E}_L}{z_1} \bar{H}_1^{\perp(1)} + \lambda_2 H_1^{\perp(1)} \frac{\bar{E}_L}{z_2} - \lambda_1 H_{1L}^{\perp(1)} \frac{\bar{E}}{z_2} - \lambda_2 \frac{\tilde{E}}{z_1} \bar{H}_{1L}^{\perp(1)} \right) \right] \Bigg\}. \quad (D.2)
\end{aligned}$$

Appendix E. Integrated twice-weighted hadron tensor

We display only the leading terms of the hadron tensor weighted with two factors $(q_T \cdot a)(q_T \cdot b)$ and integrated over q_T .

$$\begin{aligned}
\int d^2 q_T (q_T \cdot a) (q_T \cdot b) \mathcal{W}_S^{\mu\nu} &= 12e^2 z_2 z_1 \left\{ g_{\perp}^{\mu\nu} \left[-a \cdot b \left(M_1^2 D_1^{(1)} \bar{D}_1 \right. \right. \right. \\
& + M_2^2 D_1 \bar{D}_1^{(1)} - \lambda_1 \lambda_2 M_1^2 G_1^{(1)} \bar{G}_1 - \lambda_1 \lambda_2 M_2^2 G_1 \bar{G}_1^{(1)} \Big) \\
& + 2S_{1\perp} \cdot S_{2\perp} a \cdot b M_1 M_2 D_{1T}^{\perp(1)} \bar{D}_{1T}^{\perp(1)} \\
& + (a \cdot S_{1\perp} b \cdot S_{2\perp} + a \cdot S_{2\perp} b \cdot S_{1\perp}) M_1 M_2 \left(G_{1T}^{(1)} \bar{G}_{1T}^{(1)} - D_{1T}^{\perp(1)} \bar{D}_{1T}^{\perp(1)} \right) \Big] \\
& - a \cdot b \left(S_{1\perp}^{\{\mu} S_{2\perp}^{\nu\}} + g_{\perp}^{\mu\nu} S_{1\perp} \cdot S_{2\perp} \right) \\
& \times \left(M_1^2 H_1^{(1)} \bar{H}_1 + M_2^2 H_1 \bar{H}_1^{(1)} - \frac{M_1^2}{2} H_{1T}^{\perp(2)} \bar{H}_1 - \frac{M_2^2}{2} H_1 \bar{H}_{1T}^{\perp(2)} \right) \\
& - \left[a \cdot S_{1\perp} \left(S_{2\perp}^{\{\mu} b^{\nu\}} + g_{\perp}^{\mu\nu} b \cdot S_{2\perp} \right) \right. \\
& + b \cdot S_{1\perp} \left(S_{2\perp}^{\{\mu} a^{\nu\}} + g_{\perp}^{\mu\nu} a \cdot S_{2\perp} \right) \Big] \frac{M_1^2}{2} H_{1T}^{\perp(2)} \bar{H}_1 \\
& - \left[a \cdot S_{2\perp} \left(S_{1\perp}^{\{\mu} b^{\nu\}} + g_{\perp}^{\mu\nu} b \cdot S_{1\perp} \right) \right. \\
& + b \cdot S_{2\perp} \left(S_{1\perp}^{\{\mu} a^{\nu\}} + g_{\perp}^{\mu\nu} a \cdot S_{1\perp} \right) \Big] \frac{M_2^2}{2} H_1 \bar{H}_{1T}^{\perp(2)} \\
& - 2M_1 M_2 \left(a^{\{\mu} b^{\nu\}} + g_{\perp}^{\mu\nu} a \cdot b \right) \left(H_1^{\perp(1)} \bar{H}_1^{\perp(1)} + \lambda_2 \lambda_1 H_{1L}^{\perp(1)} \bar{H}_{1L}^{\perp(1)} \right) \\
& \left. + M_1 M_2 \left(a^{\{\mu} \epsilon_{\perp}^{\nu\} \rho} b_{\rho} + b^{\{\mu} \epsilon_{\perp}^{\nu\} \rho} a_{\rho} \right) \left(\lambda_1 H_{1L}^{\perp(1)} \bar{H}_1^{\perp(1)} - \lambda_2 H_1^{\perp(1)} \bar{H}_{1L}^{\perp(1)} \right) \right\} \quad (E.1)
\end{aligned}$$

and

$$\int d^2 q_T (q_T \cdot a) (q_T \cdot b) \mathcal{W}_A^{\mu\nu} = 12e^2 z_1 z_2 \left\{ -i \epsilon_{\perp}^{\mu\nu} a \cdot b \left[\lambda_2 M_2^2 D_1 \bar{G}_1^{(1)} \right. \right.$$

$$\begin{aligned}
& + \lambda_2 M_1^2 D_1^{(1)} \overline{G}_1 - \lambda_1 M_2^2 G_1 \overline{D}_1^{(1)} - \lambda_1 M_1^2 G_1^{(1)} \overline{D}_1 \Big] \\
& + i \left(S_{1\perp}^{[\mu} a^{\nu]} \mathbf{b} \cdot \mathbf{S}_{2\perp} + S_{1\perp}^{[\mu} b^{\nu]} \mathbf{a} \cdot \mathbf{S}_{2\perp} \right) M_1 M_2 D_{1T}^{\perp(1)} \overline{G}_{1T}^{(1)} \\
& + i \left(S_{2\perp}^{[\mu} a^{\nu]} \mathbf{b} \cdot \mathbf{S}_{1\perp} + S_{2\perp}^{[\mu} b^{\nu]} \mathbf{a} \cdot \mathbf{S}_{1\perp} \right) M_1 M_2 G_{1T}^{(1)} \overline{D}_{1T}^{\perp(1)} \Big\}. \tag{E.2}
\end{aligned}$$

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